Neutrino-induced electroweak symmetry breaking in supersymmetric SO(10) models

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ABSTRACT: The radiative electroweak symmetry breaking, the unification of third-generation Yukawa couplings, and flavor-changing rare decay are investigated in two types of supersymmetric SO(10) scenarios taking into account of the effects of neutrino physics, i.e. the observed large generation mixing and tiny mass scale. The first scenario is minimal, including right-handed neutrinos at intermediate scale with the unification of third-generation Yukawa couplings. Another is the case that the large mixing of atmospheric neutrinos originates from the charged-lepton sector. Under the SO(10)-motivated boundary conditions for supersymmetry-breaking parameters, typical low-energy particle spectrum is discussed and the parameter space is identified which satisfies the conditions for successful radiative electroweak symmetry breaking and the experimental mass bounds of superparticles. In particular, the predictions of the bottom quark mass and the $b \to s\gamma$ branching ratio are fully analyzed. In both two scenarios, new types of radiative electroweak symmetry breaking are achieved with the effects of neutrino couplings. The Yukawa unification becomes compatible with the bottom quark mass and the experimental constraints from flavor-violating rare processes, and the hierarchical superparticle mass spectrum is obtained.

KEYWORDS: Supersymmetric Standard Model, GUT, Neutrino Physics.

Contents

1. Introduction

For the last decades, a lot of articles have been devoted to trying to understand the underlying particle theory beyond the standard model (SM). Among them, the minimal supersymmetric standard model (MSSM) is conceived to be one of the most promising candidates for its brevity and various attractive features. A well-known example is the unification of SM gauge coupling constants at some high-energy scale [1]. The exploration of supersymmetric grand unified theory (GUT) has been therefore one of the most important subjects in particle physics. In addition to the gauge coupling unification, the GUT framework often

leads to the unification of Yukawa coupling constants. For example, in the minimal SU(5) model [2], a right-handed down-type quark and a lepton doublet belong to a single quintuplet representation of SU(5) group, and especially gives the bottom-tau Yukawa unification at high-energy scale. It is found in the MSSM that the bottom-tau unification is preferred in light of the experimentally measured values of the bottom quark and tau lepton masses. Moreover the detailed numerical analyses of renormalization-group (RG) running of gauge and Yukawa couplings have brought up a possibility of top-bottom-tau Yukawa unification with a large ratio between two vacuum expectation values (VEVs) of the MSSM Higgs doublets [3]. Hence one is naturally led to the scenario where all matter fermions of one generation are unified in a single representation of GUT group. The simplest candidate for such unified gauge group is SO(10) where the SM fermions are included in 16-dimensional representations. Various phenomenological studies of SO(10)-type Yukawa unification have been performed in the literature [4 – 11].

In the phenomenological study of Yukawa unification, one of the main subjects is the masses of third-generation fermions. It is known that the bottom quark mass is largely affected by low-energy threshold corrections which are induced via decoupling of superpartners of SM particles and bring a change of several tens of percents to the bottom quark mass estimation [12, 5 – 7]. The magnitude of the corrections depends on low-energy values of supersymmetry (SUSY) breaking parameters and supersymmetric Higgs mass parameter. Consequently the low-energy estimation of fermion masses also has strong dependence on the superparticle spectrum. It is argued from detailed analysis that the threshold correction to the bottom quark mass must be suppressed than its naively expected value so that the Yukawa unification leads to the top, bottom, and tau masses within the experimentally allowed ranges [5 – 7, 11].

In addition to the successful prediction of charged fermion masses, the recent experimental results for neutrino physics also seem to prefer the unified theory. An important property of neutrinos is their tiny mass scale compared to the other SM fermions, the smallness of which scale is naturally realized by introducing right-handed neutrinos and their large Majorana masses, called the seesaw mechanism [13]. The right-handed neutrinos are unified into 16-dimensional representations of SO(10), combined with the other SM fermions. Moreover the recent study of the solar and atmospheric neutrinos [14] has revealed that there is large generation mixing in the lepton sector, while the corresponding mixing angles in the quark sector are known to be small. Such disparate flavor structure between quarks and leptons is naively difficult to be obtained in unified theory, since as stated above the matter fermions are combined into GUT multiplets. A way to ameliorate this problem is proposed in GUT framework to naturally accommodate the large lepton mixing with asymmetric forms of Yukawa matrices, the so-called lopsided forms [15 – 20].

In GUT models with the above-mentioned neutrino property, the RG evolution of coupling constants is expected to be altered from the naive top-bottom-tau Yukawa unification by including the effects of neutrino couplings and/or the generation structure. Assuming the seesaw mechanism and taking no account of lepton large mixing, the phenomenological study of neutrino Yukawa effects on gauge and Yukawa RG evolution has been performed, e.g. in refs. [21 – 23]. The analysis has shown that the evaluation of gauge couplings and

third-generation fermion masses is slightly changed; for example, the prediction of top quark mass is made up to 3 GeV [22] and the low-energy value of the strong gauge coupling constant is decreased by a few percents [23]. A relevance of leptonic generation mixing for the bottom-tau Yukawa unification has also been discussed [24]. These RG studies however only deal with the gauge and Yukawa coupling evolutions. As stated above, the low-energy threshold corrections to the bottom quark mass often play a significant role in the analysis of fermion masses. Since the corrections are determined by superparticle mass spectrum, the inclusion of dimensionful parameters into the RG analysis is an important issue for a complete study of neutrino effects.

Concerning the RG evolution of mass parameters, the radiative electroweak symmetry breaking (EWSB) [25] should be carefully examined. The successful radiative EWSB generally restricts SUSY-breaking parameters at high-energy scale and also low-energy superparticle spectrum in the top-bottom-tau Yukawa unification. The resulting spectrum often leads to a large magnitude of low-energy threshold corrections, following which the prediction of third-generation fermion masses are difficult to be consistent with the experimentally allowed ranges [5–7, 11].

In the present work, we perform detailed analysis of radiative EWSB and third-generation fermion masses in grand unified models, taking into account the neutrino property indicated by the recent experimental results. In addition, the $b \to s\gamma$ decay rate is also evaluated since the experimental constraint on the decay rate tends to severely restrict low-energy superparticle spectrum [8, 26]. For comparison and completeness, we first review the top-bottom-tau Yukawa unification without neutrino effects, following which we will include the effects of neutrino physics in two cases; the first is to consider the Yukawa unification of top, bottom, tau, and third-generation neutrino where no other (small) elements in Yukawa matrices are involved, and in the second case, the large generation mixing in the lepton sector is included in the analysis by assuming the lopsided form of Yukawa matrices. In both cases, we introduce large Majorana masses for right-handed neutrinos, and tiny neutrino masses are induced through the seesaw mechanism. Since low-energy threshold correction to the bottom quark mass is rather sensitive to SUSY-breaking parameters, the running bottom quark mass is treated as an output parameter in order to reveal the behavior of threshold corrections. On the other hand, the physical top quark mass is used as an input quantity. In this paper we adopt the value $m_t^{\text{pole}} = 178\,\text{GeV}$ [27] and give some comments on the case of the most recent report on the top quark mass $m_t^{\text{pole}} = 172.7\,\text{GeV}$ [28].

The paper is organized as follows. In section 2 we present an overview of the third-generation fermion masses, the $b \to s\gamma$ constraint, and radiative EWSB in Yukawa unification scenario where low-energy SUSY-breaking quantities are treated as free parameters. This treatment reveals preferred types of low-energy superparticle spectrum and provides an useful reference for later discussions of radiative EWSB in specific models. In section 3 the phenomenology of the top-bottom-tau Yukawa unification is discussed. Sections 4 and 5 include the neutrino effects into the analysis of EWSB; with the Yukawa unification of top-bottom-tau and third-generation neutrino in section 4 and with the lopsided form of mass textures in section 5. We summarize the results in section 6.

2. General aspects in Yukawa unification

In this section, we present an overview of low-energy phenomenology in general Yukawa unification scenario; the third-generation fermion masses, the $b \to s\gamma$ decay, and the EWSB are discussed. We assume the gauge coupling unification and also Yukawa coupling unification at the GUT scale. The low-energy values of these dimensionless couplings are determined by solving RG equations. On the other hand, there is no assumption about GUT models and dimensionful parameters at high-energy regime. Accordingly, low-energy SUSY-breaking parameters are treated as independent variables. The general analysis given here reveals preferred types of low-energy spectrum and would be useful for later discussion in specific high-energy models with neutrino effects.

Below the GUT scale, the theory is assumed to be the MSSM whose superpotential is given by

$$W_{\text{MSSM}} = Q_i(Y_u)_{ij}\bar{u}_j H_u + Q_i(Y_d)_{ij}\bar{d}_j H_d + L_i(Y_e)_{ij}\bar{e}_j H_d + \mu H_u H_d, \qquad (2.1)$$

where Q_i, $\bar{u}_i$, $\bar{d}_i$, L_i, $\bar{e}_i$ and H_u, H_d are three-generation matter superfields and Higgs superfields, respectively. The 3×3 complex matrices Y_u, Y_d, Y_e with generation indices mean Yukawa coupling constants and μ is the supersymmetric Higgs mass parameter. We assume the Yukawa coupling unification in that the 3-3 elements of Yukawa matrices, i.e. top, bottom, and tau Yukawa couplings, y_t, y_b and y_τ, are unified at high-energy scale:

$$y_t(M_G) = y_b(M_G) = y_\tau(M_G) \equiv y_G, \qquad (2.2)$$

where $M_G \simeq 10^{16}$ GeV is the GUT scale. The precise definition of M_G will be given in the next section.

2.1 Top, bottom, tau masses in Yukawa unification

We first review the RG evolution of third-generation Yukawa couplings in Yukawa unification scenario. It is found that the Yukawa unification is compatible with the observed values of heavy fermion masses, while the low-energy prediction of mass eigenvalues is rather sensitive to superparticle spectrum [5, 7, 11]. In the following analysis, the exact top-bottom-tau Yukawa coupling unification is assumed at the GUT scale and other small entries in Yukawa matrices are safely neglected. Once the unified gauge coupling $g_G = g_1(M_G) = g_2(M_G) = g_3(M_G)$ and the unified Yukawa coupling y_G are fixed, one can evaluate with the MSSM RG equations the $\overline{\text{DR}}$ running gauge and Yukawa couplings at the Z-boson mass scale M_Z. The Yukawa couplings and the third-generation $\overline{\text{DR}}$ running fermion masses are related as

$$\begin{aligned}
m_t^{\overline{\text{DR}}}(M_Z) &= v_H \sin\beta \, y_t^{\overline{\text{DR}}}(M_Z)\,(1 + \Delta_t), \\
m_b^{\overline{\text{DR}}}(M_Z) &= v_H \cos\beta \, y_b^{\overline{\text{DR}}}(M_Z)\,(1 + \Delta_b), \\
m_\tau^{\overline{\text{DR}}}(M_Z) &= v_H \cos\beta \, y_\tau^{\overline{\text{DR}}}(M_Z)\,(1 + \Delta_\tau),
\end{aligned} \qquad (2.3)$$

where v_H parametrizes the $\overline{\text{DR}}$ Higgs VEV at the electroweak scale which is taken as $v_H = 174.6$ GeV. The angle β is defined by the VEV ratio of two Higgs doublets as

$\tan\beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$. The low-energy threshold corrections from heavy superparticles [29], Δ_t, Δ_b and Δ_τ, have been included. The Yukawa unification at high-energy scale and the MSSM RG evolution generally lead to almost the same size of top and bottom Yukawa couplings, $y_t^{\overline{\rm DR}}(M_Z) \simeq y_b^{\overline{\rm DR}}(M_Z)$, and consequently the ratio $\tan\beta$ should be very large to attain the observed mass hierarchy between the top and bottom quarks. The required value of $\tan\beta$ is roughly estimated as

$$\tan\beta \simeq \frac{m_t^{\overline{\rm DR}}(M_Z)}{m_b^{\overline{\rm DR}}(M_Z)} \simeq \mathcal{O}(50-60), \tag{2.4}$$

if one neglects threshold corrections and small differences between the top and bottom Yukawa couplings generated through the RG evolution down to low-energy regime.

In such a large $\tan\beta$ case, low-energy threshold corrections from heavy superparticles become important [5, 6, 29]. The sizes of corrections to gauge couplings and top quark mass Δ_t mainly depend on the overall scale of superparticle masses. On the other hand, the correction to bottom quark mass Δ_b is controlled by a ratio between SUSY-breaking masses and the μ parameter. This is a consequence of the fact that Δ_b is dominated by finite parts of threshold corrections which are approximately constituted of the following two contributions [5, 6]:

$$\Delta_b^{\tilde{g}} \simeq \frac{2\tan\beta}{3\pi}\left(\frac{g_3^2}{4\pi}\right)\mu M_3\, I(M_3^2, m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2), \tag{2.5}$$

$$\Delta_b^{\tilde{\chi}^+} \simeq \frac{\tan\beta}{4\pi}\left(\frac{y_t^2}{4\pi}\right)\mu A_t\, I(\mu^2, m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2), \tag{2.6}$$

where

$$I(x,y,z) = \frac{xy\ln(y/x) + yz\ln(z/y) + zx\ln(x/z)}{(x-y)(y-z)(z-x)}. \tag{2.7}$$

The corrections $\Delta_b^{\tilde{g}}$ and $\Delta_b^{\tilde{\chi}^+}$ denote the contributions to Δ_b from gluino-scalar bottom and chargino-scalar top loop diagrams. The SUSY-breaking couplings M_3, A_t, $m_{\tilde{t}_{1,2}}$, $m_{\tilde{b}_{1,2}}$ are the gluino mass, the trilinear coupling of scalar top quark, and the mass eigenvalues of top and bottom scalar quarks, respectively. The complete form of corrections including generation mixing is found, e.g. in [30]. The loop function $I(x,y,z)$ behaves as $\mathcal{O}(1/M)$ with M being the maximum value among x, y, and z. If there is no large hierarchy among SUSY-breaking mass parameters and μ, the corrections $|\Delta_b^{\tilde{g}}|$ and $|\Delta_b^{\tilde{\chi}^+}|$ are expected to become $\mathcal{O}(1)$ for a large value of $\tan\beta$. Also Δ_τ is dominated by finite parts in the large $\tan\beta$ case, but it is generally smaller than Δ_b since the large gauge and Yukawa couplings are absent in the expression of Δ_τ. In the Yukawa unification scenario, Δ_t and Δ_τ roughly become $|\Delta_t|$, $|\Delta_\tau| \lesssim \mathcal{O}(0.1)$ when superparticles are lighter than a few TeV [29].

In this section we take SUSY-breaking parameters as free variables and let the threshold corrections Δ_i represent low-energy superparticle spectrum. In other words, once a preferred range of Δ_i is found, that in turn gives a constraint on low-energy SUSY-breaking parameters. For fixed values of low-energy gauge couplings, top-quark and tau-lepton masses, and the threshold corrections $\Delta_{t,\tau}$, the Yukawa unification hypothesis predicts
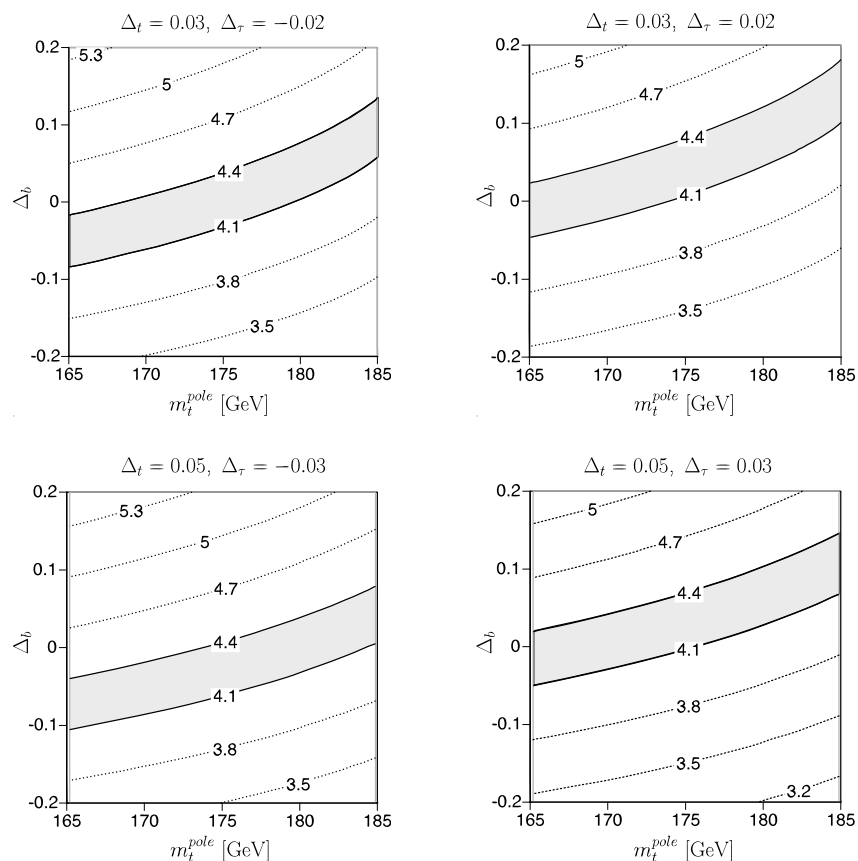
the low-energy bottom quark mass as a function of Δ_b. The allowed range of Δ_b is evaluated by the following numerical procedure. First the input parameters are the $\overline{\text{MS}}$ gauge couplings $g_1^{\overline{\text{MS}}}(M_Z)$ and $g_2^{\overline{\text{MS}}}(M_Z)$ given in [31], $\alpha_3^{\overline{\text{MS}}}(M_Z) = 0.1187$ ($\alpha_3 = g_3^2/4\pi$), $M_Z = 91.1876\,\text{GeV}$ and $m_\tau^{\text{pole}} = 1776.99\,\text{MeV}$ [27]. The prediction of bottom-quark mass is correlated to the input value of top-quark mass, which is varied in the range $165\,\text{GeV} < m_t^{\text{pole}} < 185\,\text{GeV}$. At the scale M_Z, the $\overline{\text{MS}}$ gauge couplings are converted to the $\overline{\text{DR}}$ ones including low-energy corrections of suitable range. The two-loop MSSM RG equations [32] are traced from M_Z up to the scale where the $\overline{\text{DR}}$ $\text{SU}(2)_L$ and $\text{U}(1)_Y$ (in GUT normalization) gauge couplings meet. That defines the GUT scale M_G and the unified gauge coupling $g_G = g_1^{\overline{\text{DR}}}(M_G) = g_2^{\overline{\text{DR}}}(M_G)$. At the GUT scale, the exact top-bottom-tau Yukawa unification is imposed. The Yukawa couplings for the first and second generations are neglected:

$$Y_u^{\overline{\text{DR}}}(M_G) = Y_d^{\overline{\text{DR}}}(M_G) = Y_e^{\overline{\text{DR}}}(M_G) = \begin{pmatrix} & & \\ & & \\ & & y_G \end{pmatrix}. \tag{2.8}$$

For fixed values of Δ_t and Δ_τ, the unified Yukawa coupling y_G has a one-to-one correspondence to m_t^{pole} and can be determined. The procedure is as follows. The two-loop MSSM RG evolution from M_G down to M_Z determines $y_t^{\overline{\text{DR}}}(M_Z)$, $y_b^{\overline{\text{DR}}}(M_Z)$ and $y_\tau^{\overline{\text{DR}}}(M_Z)$, and then with an input value of tau-lepton mass and assumed Δ_τ, a required value of $\tan\beta$ is found, where $m_\tau^{\overline{\text{DR}}}(M_Z)$ is extracted from m_τ^{pole} using the SM corrections [31]. Once $\tan\beta$ is fixed, m_t^{pole} is evaluated by $y_t^{\overline{\text{DR}}}(M_Z)$, v_H, assumed Δ_t, and the SM QCD contribution [29]. A series of the above calculations is reiterated by varying y_G until to achieve the input value of m_t^{pole}. Finally, the prediction of $m_b^{\overline{\text{DR}}}(M_Z)$ is derived as a function of m_t^{pole} and Δ_b, and is convert to $m_b^{\overline{\text{MS}}}(m_b)$ taken into account the two-loop SM QCD corrections [5]. The prediction of $m_b^{\overline{\text{MS}}}$ depends on other input parameters besides the top-quark mass and Δ_b. We will mention this later.

Figure 1 shows $m_b^{\overline{\text{MS}}}(m_b)$ as the function of m_t^{pole} and Δ_b. In the figures, we set Δ_t and Δ_τ to typical values which roughly correspond to the case that superparticle masses are less than a few TeV. The observed value of bottom-quark mass is given by $m_b^{\overline{\text{MS}}}(m_b) = 4.1-4.4\,\text{GeV}$ [27] and shown as shaded regions in the figures. It is immediately found that the threshold correction $|\Delta_b|$ must be small; $|\Delta_b| \lesssim 0.1$ in a wide range of top-quark mass, which result is consistent with the previous analysis [7, 11]. The result is slightly changed by varying input parameters. For example, $m_b^{\overline{\text{MS}}}(m_b)$ is altered $\pm 0.2\,\text{GeV}$ with $\alpha_3^{\overline{\text{MS}}}(M_Z)$ in the range $0.116-0.121$ for fixed values of m_t^{pole} and Δ_b. We also checked the dependence of $m_b^{\overline{\text{MS}}}(m_b)$ on unknown GUT-scale threshold corrections to Yukawa couplings. If a five percents deviation of $y_b^{\overline{\text{DR}}}(M_G)$ or $y_\tau^{\overline{\text{DR}}}(M_G)$ from y_G is included, $|m_b^{\overline{\text{MS}}}(m_b)|$ typically has an ambiguity less than $0.3\,\text{GeV}$. The GUT-scale correction to $y_t^{\overline{\text{DR}}}(M_G)$ is turned out to be irrelevant.

Such a small value of experimentally preferred $|\Delta_b|$ indicates that there must be some hierarchical structure among SUSY-breaking mass parameters and μ parameter, otherwise, two types of large threshold corrections (2.5) and (2.6) must cancel out to each other. The latter case largely depends on the detail of superparticle spectrum and is

Figure 1: Typical predictions of bottom-quark mass in the Yukawa unification. The low-energy value $m_b^{\overline{\text{MS}}}(m_b)$ is shown as the function of top-quark mass and threshold correction Δ_b. The input parameters are taken as the $\overline{\text{MS}}$ gauge couplings $\alpha_1^{\overline{\text{MS}}}(M_Z) = 0.01698$ and $\alpha_2^{\overline{\text{MS}}}(M_Z) = 0.03364$, $\alpha_3^{\overline{\text{MS}}}(M_Z) = 0.1187$ $(\alpha_i = g_i^2/4\pi)$, $M_Z = 91.1876\,\text{GeV}$ and $m_\tau^{\text{pole}} = 1776.99\,\text{MeV}$. The threshold corrections to top-quark and tau-lepton masses are, as examples, set to be $\Delta_t = 0.03$ and $\Delta_\tau = -0.02$ in the left-top, $\Delta_t = 0.03$ and $\Delta_\tau = 0.02$ in the right top, $\Delta_t = 0.05$ and $\Delta_\tau = -0.03$ in the left bottom, and $\Delta_t = 0.05$ and $\Delta_\tau = 0.03$ in the right-bottom figures, respectively.

therefore a model-dependent option. As for the former case, however, an interesting resolution is found. It can be seen from the expression (2.5) and (2.6) that, a small correction $|\Delta_b|$ is obtained if scalar quark masses are relatively larger than the other mass parameters; the gluino mass, scalar top trilinear coupling, and μ parameter. It was pointed out in [5] that such a hierarchical mass pattern does follow from symmetry argument: the Peccei-Quinn (PQ) symmetry [33] which rotates Higgs particles suppresses the μ parameter, and the R symmetry which acts on the fermionic coordinate forbids holomorphic SUSY-breaking parameters in the symmetric limit. Therefore if approximate PQ and R symmetries are realized in the theory, the Yukawa unification hypothesis predicts experimentally-allowed masses for the third-generation fermions (top, bottom quarks and tau lepton) [5, 7, 11].

2.2 $b \rightarrow s\gamma$ decay

As stated previously, the Yukawa unification generally requires a large value of $\tan \beta$. It is well known that in the large $\tan \beta$ case the experimental observation of $b \rightarrow s\gamma$ decay provides severe constraints on low-energy superparticle spectrum [6 – 8, 26]. In this subsection we shortly review the characteristic feature of $b \rightarrow s\gamma$ constraint in the large $\tan \beta$ case.

The experimentally-measured branching ratio of the $b \rightarrow s\gamma$ decay is $\mathcal{B}(b \rightarrow s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$ [34], and the SM prediction for the branching ratio is theoretically in excellent agreement with the experimental observation [35]. In the MSSM, the $b \rightarrow s\gamma$ decay amplitude consists of several loop diagrams: up-type quark-W boson (A_{SM}), up-type quark-charged Higgs boson (A_{H^+}), up-type scalar quark-chargino ($A_{\tilde{\chi}^+}$), down-type scalar quark-neutralino ($A_{\tilde{\chi}^0}$), and down-type scalar quark-gluino ($A_{\tilde{g}}$). The magnitudes of these amplitudes depend on masses of fields running in the internal loops. The amplitude A_{H^+} always gives a constructive contribution to the W-boson loop A_{SM}, which together make up the standard model contribution, while the other contributions $A_{\tilde{\chi}^+}$, $A_{\tilde{\chi}^0}$, and $A_{\tilde{g}}$ are constructive or destructive, depending on the signs of scalar trilinear couplings and μ parameter. It is known that $A_{\tilde{\chi}^+}$, $A_{\tilde{\chi}^0}$, and $A_{\tilde{g}}$ scale as $\tan \beta$ and are amplified with Yukawa unification [8]. In most cases, since $A_{\tilde{\chi}^+}$ is largest among the superparticle-induced contributions, we focus on $A_{\tilde{\chi}^+}$ in the following qualitative discussion, for simplicity. In numerical evaluation, all the decay modes are included according to the formulas in [36].

In the Yukawa unification scenario, $\tan \beta$ is large and the chargino contribution $A_{\tilde{\chi}^+}$ becomes important. The decay rate of $b \rightarrow s\gamma$ is therefore sensitive to the sign of $A_{\tilde{\chi}^+}$. It is found that the sign of $A_{\tilde{\chi}^+}$ is determined by that of μ in the minimal supergravity scenario where scalar trilinear coupling is negative at low-energy regime. For a positive μ parameter, $A_{\tilde{\chi}^+}$ gives a destructive contribution to the standard model prediction. If $A_{\tilde{\chi}^+}$ and A_{H^+} cancel each other out, a relatively light superparticle spectrum is experimentally allowed [8, 26]. The degree of such cancellation is roughly controlled by mass ratio among the charged Higgs boson, charginos, and up-type scalar quarks. On the other hand, for a negative μ parameter, $A_{\tilde{\chi}^+}$ gives a constructive contribution to the standard model prediction. Considering the fact that the standard model contribution is well fitted to the experimental result, new physics contribution must be suppressed. In particular, superparticle spectrum with a negative μ parameter is highly constrained from a viewpoint of $b \rightarrow s\gamma$ process.

Figure 2 shows typical results for the $b \rightarrow s\gamma$ branching ratio in the large $\tan \beta$ case. The vertical axis means the mass of charged Higgs boson and the horizontal one $M_{\tilde{C}}$ which characterizes superparticle masses (note that, in this section, SUSY-breaking parameters are free variables). As an example to discuss qualitative feature, we set in the figures the masses of colored superparticles and $|\mu|$ to be $M_{\tilde{C}}$, and those of uncolored ones, trilinear couplings of scalar top and bottom to be $M_{\tilde{C}}/2$. The lower limits of M_{H^+} and $M_{\tilde{C}}$ come from the experimental mass bounds on charged Higgs boson and gluino: $M_{H^+} > 79.3 \, \mathrm{GeV}$ and $M_3 > 195 \, \mathrm{GeV}$ (95% CL) [27]. The set of other input parameters is shown in the figure caption. Since there may be some theoretical ambiguities in the estimation, we take in this
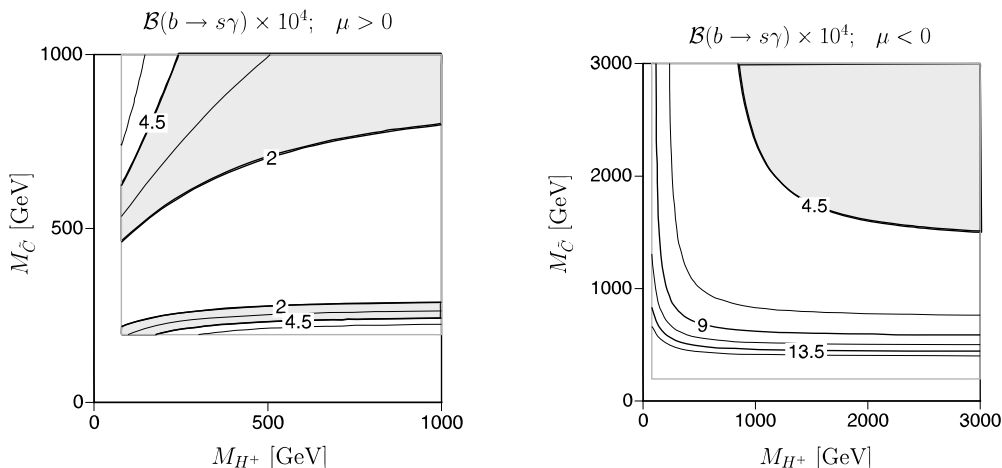
Figure 2: Typical results for $b \to s\gamma$ branching ratio in the large $\tan\beta$ case ($\tan\beta = 50$). The two axes denote the masses of charged Higgs boson and colored superparticles. The sign of μ is set to be positive (negative) in the left (right) figure. The input parameters besides those in figure 1 are the W-boson mass $M_W^{\text{pole}} = 80.425\,\text{GeV}$, $m_b^{\overline{\text{MS}}}(m_b) = 4.25\,\text{GeV}$, $m_t^{\text{pole}} = 178\,\text{GeV}$, $m_u^{\overline{\text{MS}}}(M_Z)/m_t^{\overline{\text{MS}}}(M_Z) = 8.6 \times 10^{-7}$, $m_c^{\overline{\text{MS}}}(M_Z)/m_t^{\overline{\text{MS}}}(M_Z) = 3.7 \times 10^{-3}$, $m_d^{\overline{\text{MS}}}(M_Z)/m_b^{\overline{\text{MS}}}(M_Z) = 1.0 \times 10^{-3}$, $m_s^{\overline{\text{MS}}}(M_Z)/m_b^{\overline{\text{MS}}}(M_Z) = 2.2 \times 10^{-2}$, and the observed values of the generation mixing matrix elements.

paper a rather conservative constraint on the $b \to s\gamma$ branching ratio

$$2.0 \times 10^{-4} < \mathcal{B}(b \to s\gamma) < 4.5 \times 10^{-4}. \qquad (2.9)$$

The region which satisfies this constraint is shaded in the figures.

It is found from figure 2 that the $b \to s\gamma$ branching ratio shows quite different behavior between the positive and negative values of μ parameter. In the $\mu > 0$ case, there exist large parameter regions which satisfy the experimental constraint. The branching ratio becomes larger in the regions $M_{H^+} \ll M_{\tilde{C}}$ and $M_{H^+} \gg M_{\tilde{C}}$. That is a consequence of the fact that $A_{\tilde{\chi}^+}$ gives a destructive contribution to A_{SM} and A_{H^+}. For $300\,\text{GeV} \lesssim M_{\tilde{C}} \lesssim 500\,\text{GeV}$, $A_{\tilde{\chi}^+}$ has a similar magnitude (and opposite sign) of the sum of A_{SM} and A_{H^+}, and the branching ratio almost vanishes. In the region where $M_{\tilde{C}}$ is closer to its experimental lower bound, the branching ratio takes a larger value, which is dominated by superparticle loop diagrams. In this way, in order to satisfy the $b \to s\gamma$ constraint with a positive μ parameter, the masses of charged Higgs boson and superparticles must lie in almost the same order to realize a cancellation among the partial amplitudes, or both the charged Higgs boson and superparticles are heavy as much as $1\,\text{TeV}$. On the other hand, for $\mu < 0$, the $b \to s\gamma$ process severely restricts the charged Higgs boson and superparticle masses. Neither M_{H^+} nor $M_{\tilde{C}}$ is allowed to be smaller than a few TeV.

To summarize this subsection, in the large $\tan\beta$ case as in the Yukawa unification, the observation of $b \to s\gamma$ decay provides severe constraints on low-energy superparticle spectrum. For $\mu > 0$, since a cancellation among different contributions is possible, a lighter superparticle spectrum is allowed when the masses of charged Higgs boson and

superparticles lie in a similar order. For $\mu < 0$, various contributions are additive and make up a large branching ratio, and hence light superparticles are disfavored; to have branching ratio within experimentally-allowed range, both the charged Higgs boson and superparticles must be heavier than a few TeV.

2.3 MSSM Higgs potential

In the large $\tan\beta$ case, successful EWSB requires several conditions among mass parameters in the theory. In particular, the μ parameter and CP-odd neutral Higgs boson mass are related to SUSY-breaking mass parameters of up- and down-type Higgs fields. Such conditions should be satisfied at the electroweak scale and are promoted to those for GUT-scale parameters. The detailed discussions of radiative EWSB for specific high-energy models will be given in later sections.

In the MSSM, the tree-level Higgs potential takes the following form:

$$
\begin{aligned}
V_{\text{Higgs}} = {}& (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\
& + \left[B\mu(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.} \right] \\
& + \frac{g_2^2 + g'^2}{8}\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 + \frac{g_2^2}{2}\left|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}\right|^2
\end{aligned}
\tag{2.10}
$$

where $H_u = (H_u^+, H_u^0)^{\text{T}}$, $H_d = (H_d^0, H_d^-)^{\text{T}}$ are the lowest components of Higgs superfields, $m_{H_{u,d}}^2$ are the SUSY-breaking mass parameters for up- and down-type Higgs scalars, and B is the SUSY-breaking Higgs mixing parameter. The coupling g' is $\text{U}(1)_Y$ gauge coupling which is related to g_1 in GUT normalization as $g' = \sqrt{3/5}\,g_1$. All the couplings are running parameters defined in the $\overline{\text{DR}}$ scheme, but for notational simplicity, the subscripts "$\overline{\text{DR}}$" will not be explicitly shown in the below.

When the Higgs scalars develop non-vanishing VEVs, one of VEVs of the charged scalars is always set to be zero with $\text{SU}(2)_L$ gauge transformation, and another charged one becomes zero due to the minimization condition. Therefore the Higgs potential V_{Higgs} induces the correct pattern of EWSB; $\text{SU}(2)_L \times \text{U}(1)_Y \to \text{U}(1)_{\text{EM}}$. Also using the $\text{U}(1)_Y$ gauge transformation and a phase rotation of H_u and H_d, the VEVs of neutral components can be made real and positive. Thus the Higgs VEVs are parametrized by real, positive parameters as

$$
\langle H_u \rangle = \begin{pmatrix} 0 \\ v_H \sin\beta \end{pmatrix}, \qquad \langle H_d \rangle = \begin{pmatrix} v_H \cos\beta \\ 0 \end{pmatrix},
\tag{2.11}
$$

where $0 \le \beta \le \pi/2$. In the vacuum, the Z-boson mass M_Z is given by $M_Z^2 = \frac{1}{2}(g_2^2 + g'^2)v_H^2$. The nonzero and finite VEVs are obtained when the mass parameters in the potential satisfy the following inequalities [25]:

$$
(m_{H_u}^2 + |\mu|^2)(m_{H_d}^2 + |\mu|^2) - |B\mu|^2 < 0
\tag{2.12}
$$

and

$$
(m_{H_u}^2 + |\mu|^2) + (m_{H_d}^2 + |\mu|^2) - 2|B\mu| > 0
\tag{2.13}
$$

at the renormalization scale $Q \sim M_{\text{SUSY}}$ which is a typical mass scale of SUSY-breaking parameters. The former condition implies the origin of field space is made unstable to have

broken electroweak symmetry, and the latter one lifts up the flat direction with $\tan \beta = 1$, otherwise the potential is unbounded from below in that direction. At the vacuum of potential, the stationary conditions are written as

$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - |\mu|^2, \tag{2.14}$$

$$\tan \beta + \cot \beta = \frac{m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2}{|B\mu|}, \tag{2.15}$$

at the classical level. The MSSM RG equations allow us to freely choose low-energy μ and B with the freedom of GUT-scale μ and B without affecting any other SUSY-breaking parameters. Thus the above stationary conditions are solved for μ and B. If the Z-boson mass and a required value of $\tan \beta$ cannot be fitted by μ and B, a desired pattern of EWSB does not occur.

In the large $\tan \beta$ limit, the stationary conditions are approximately rewritten in the following simple form:

$$M_Z^2 \simeq -2(m_{H_u}^2 + |\mu|^2), \tag{2.16}$$

$$\tan \beta \simeq \frac{m_{H_d}^2 + m_{H_u}^2 + 2|\mu|^2}{|B\mu|}. \tag{2.17}$$

Then the constraints on the Higgs mass parameters read

$$|\mu|^2 \simeq -m_{H_u}^2 - \frac{M_Z^2}{2} > 0, \tag{2.18}$$

$$|B\mu| \tan \beta \simeq m_{H_d}^2 - m_{H_u}^2 - M_Z^2 > 0. \tag{2.19}$$

The Higgs SUSY-breaking mass parameters must satisfy the inequalities (2.18) and (2.19) so that the observed Z-boson mass and a positively definite value of $\tan \beta$ are realized by consistently choosing μ and B. It is interesting to note that the left-handed sides of these inequalities are related to mass squareds of physical particles in the symmetry broken phase. In (2.18), $|\mu|^2$ is relevant to tree-level masses of charginos and neutralinos. Also the inequality (2.19) is concerned with the tree-level mass eigenvalue of CP-odd neutral Higgs boson:

$$M_A^2 = m_{H_u}^2 + m_{H_d}^2 + 2|\mu|^2 \simeq m_{H_d}^2 - m_{H_u}^2 - M_Z^2. \tag{2.20}$$

The current experimental mass bounds of the lightest neutralino $\tilde{\chi}_1^0$, the lighter chargino mass eigenstate $\tilde{\chi}_1^+$, and the CP-odd neutral Higgs boson are $m_{\tilde{\chi}_1^0} > 46 \, \text{GeV}$, $m_{\tilde{\chi}_1^+} > 67.7 \, \text{GeV}$, and $M_A > 90.4 \, \text{GeV}$ (95% CL), respectively [27]. Therefore (2.18) and (2.19) actually impose severer constraints on the Higgs mass parameters. Moreover, in the large $\tan \beta$ case, one-loop corrections to M_A and μ generally tend to be large, which is mainly due to the tadpole contribution to one-loop effective potential. In the numerical analysis in later sections, we take into account of these mass bounds and one-loop corrections.

The mass bound on M_A^2 (2.19) implies that, at low-energy regime, two Higgs SUSY-breaking masses $m_{H_u}^2$ and $m_{H_d}^2$ must be separated and the down-type Higgs mass squared

must be larger than the up-type Higgs mass squared. If $\tan\beta$ is small, a larger value of the top Yukawa coupling than the bottom (and tau) ensures such a mass separation via the RG evolution down to low energy, and successful EWSB is radiatively achieved [25]. However for the large $\tan\beta$ case as in the Yukawa unification, the top and bottom Yukawa couplings are of almost same order of magnitude throughout the RG evolution, and the splitting of two Higgs masses is not guaranteed. As will be seen in the following sections, if SO(10)-like GUT models are assumed, this mass bound of CP-odd neutral Higgs boson excludes large regions of high-energy SUSY-breaking parameter space.

3. SO(10) unification

In this section we discuss the minimal SO(10)-type GUT scenario which is a naive high-energy realization of the Yukawa unification. Contrary to the analysis in the previous section, SUSY-breaking parameters at the GUT scale are assumed to be unified, that is, a top-down approach to low-energy SUSY phenomenology. The radiative EWSB and various other aspects in this class of unification scenario have been widely investigated in the literature [6–8, 37, 38, 9, 10]. We focus here on the study of radiative EWSB, third-generation fermion masses, and $b \to s\gamma$ process for comparison with later discussions.

The model we now consider is specified by the following assumptions: (i) SO(10) gauge symmetry is broken down to the SM gauge group at the GUT scale M_G, below which the theory is just the MSSM. (ii) The MSSM matter and Higgs superfields are included in SO(10) multiplets 16_i (matter) and 10_H (Higgs), respectively. (iii) The MSSM Yukawa terms (2.1) come from a GUT-scale superpotential $W = 16_i Y_{ij} 16_j 10_H$. The Yukawa matrix Y has a large $\mathcal{O}(1)$ component $Y_{33} \equiv y_G$, and therefore the top, bottom, and tau Yukawa couplings are unified at the GUT scale. (iv) SUSY-breaking terms also respect the SO(10) symmetry. Thus the independent SUSY-breaking parameters at the GUT scale are

$$m_{16}^2,\ m_{10}^2,\ M_{1/2},\ A_0,\ B_0, \tag{3.1}$$

where m_{16}^2 (m_{10}^2) denotes the matter (Higgs) scalar masses, $M_{1/2}$ the universal gaugino mass parameter, and A_0 the universal scalar trilinear coupling. As mentioned before, the B parameter is determined at the electroweak scale by solving the EWSB conditions and hence the high-energy boundary value B_0 is irrelevant to the analysis in this paper. The boundary values of SUSY-breaking parameters in the MSSM are matched to the GUT-scale independent parameters as

$$m_{\tilde{Q}}^2(M_G)_{ij} = m_{\tilde{u}}^2(M_G)_{ij} = m_{\tilde{d}}^2(M_G)_{ij} = m_{\tilde{L}}^2(M_G)_{ij} = m_{\tilde{e}}^2(M_G)_{ij} = m_{16}^2\,\delta_{ij}, \tag{3.2}$$

$$m_{H_u}^2(M_G) = m_{H_d}^2(M_G) = m_{10}^2, \tag{3.3}$$

$$M_1(M_G) = M_2(M_G) = M_3(M_G) = M_{1/2}, \tag{3.4}$$

$$A_u(M_G)_{ij} = A_d(M_G)_{ij} = A_e(M_G)_{ij} = A_0\,\delta_{ij}, \tag{3.5}$$

where $m_{\tilde{Q}}^2$, $m_{\tilde{u}}^2$, $m_{\tilde{d}}^2$, $m_{\tilde{L}}^2$, $m_{\tilde{e}}^2$, A_u, A_d, A_e are the MSSM matter scalar masses and trilinear couplings in the generation space, respectively, and M_1, M_2, M_3 denote the SM gaugino

masses. Note that even if the universal scalar masses are assumed at a cutoff scale, RG effects above M_G might induce some difference between m_{10}^2 and m_{16}^2 at M_G [39], whose size of splitting is however rather model dependent, e.g. sensitive to GUT-breaking Higgs sector. Thus m_{10}^2 and m_{16}^2 can be taken as independent variables, and throughout this paper we equivalently use the following notation:

$$\tilde{m}_0^2 = \frac{m_{10}^2 + m_{16}^2}{2}, \qquad \xi = \frac{m_{10}^2 - m_{16}^2}{m_{10}^2 + m_{16}^2}, \tag{3.6}$$

in other words,

$$m_{10}^2 = \tilde{m}_0^2(1 + \xi), \qquad m_{16}^2 = \tilde{m}_0^2(1 - \xi). \tag{3.7}$$

In the following, we first discuss about the radiative EWSB and then study the parameter space allowed by third-generation fermion masses and the $b \to s\gamma$ observation. As seen in the previous section, the Yukawa unification hypothesis leads to third-generation fermion masses strongly correlated to the threshold correction Δ_b, which has large dependence on SUSY-breaking mass and μ parameters. Since these parameters are severely restricted by the EWSB conditions and the $b \to s\gamma$ constraint, it is a non-trivial issue whether the above type of SO(10) unification is phenomenologically viable. We will present a detailed analysis on this subject in the below.

3.1 Radiative EWSB in SO(10) unification

To have an insight about low-energy superparticle spectrum in the SO(10)-type unification, we start to discuss about the radiative EWSB. Since the Yukawa coupling unification leads to a large value of $\tan\beta$, the EWSB conditions are now approximately given by the simple form (2.18) and (2.19). In this case, as discussed before, the experimental lower bounds on M_A and $|\mu|$ strongly restrict low-energy values of mass parameters in the Higgs potential, which are promoted to the constraints on GUT-scale parameters through the RG evaluation. The SUSY-breaking mass parameters at low energy, in particular, for the Higgs fields, are calculated from the boundary values (3.2)-(3.5) with the two-loop MSSM RG equations for gauge and Yukawa couplings and the one-loop ones for dimensionful parameters. By solving the minimization conditions of the electroweak Higgs potential, we then obtain M_A and $|\mu|$ which can be generally expressed in terms of the boundary values of SUSY-breaking mass parameters and the Z-boson mass:

$$M_A^2 = (c_{ms} + c_{md}\xi)\tilde{m}_0^2 + c_M M_{1/2}^2 + c_{AM}A_0 M_{1/2} + c_A A_0^2 - M_Z^2, \tag{3.8}$$

$$|\mu|^2 = (d_{ms} + d_{md}\xi)\tilde{m}_0^2 + d_M M_{1/2}^2 + d_{AM}A_0 M_{1/2} + d_A A_0^2 - \frac{M_Z^2}{2}. \tag{3.9}$$

The coefficients c's and d's are dimensionless quantities which are numerically determined by solving the RG equations. Typical behaviors of c's and d's at $1\,\mathrm{TeV}$ are shown in figure 3 as the function of m_t^{pole}, where the procedure of numerical analysis is the same as in figure 1, $\tan\beta$ is fixed by the tau Yukawa evaluation, and the threshold corrections to top and tau-lepton masses are set to $\Delta_t = 0.03$ and $\Delta_\tau = -0.02$ as an example. Several implications of the above RG solutions are investigated below in order.
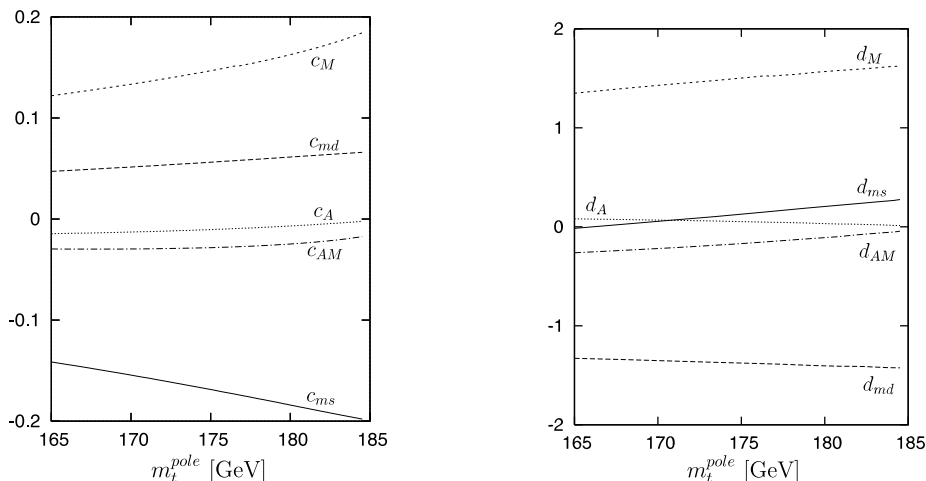
Figure 3: Typical behaviors of numerical coefficients c's and d's in the MSSM RG solutions (3.8) and (3.9) with the EWSB conditions evaluated at the renormalization scale $Q = 1\,\text{TeV}$. In the figures, the input parameters and the procedure of numerical analysis are the same as in figure 1 and the threshold corrections to top and tau-lepton masses are set to $\Delta_t = 0.03$ and $\Delta_\tau = -0.02$.

First, let us study the CP-odd neutral Higgs mass M_A (3.8). As discussed previously, the positiveness of M_A^2, i.e. a successful EWSB, requires the separation of Higgs SUSY-breaking mass parameters, $m_{H_u}^2 < m_{H_d}^2$, at the electroweak scale. In the present SO(10)-type unification where the two Higgs masses are unified at the GUT scale, the EWSB must be triggered purely by radiative corrections. It is found from figure 3 that the contributions of gaugino mass c_M and of scalar mass c_{ms} have similar magnitudes with opposite signs, while the other effects are relatively small. This suggests that a difference between up- and down-type Higgs masses is not generated in the RG evolution due to the structure of Yukawa couplings: the GUT-scale Yukawa unification means not only the top but also the bottom and tau Yukawa couplings are as large as $\mathcal{O}(1)$ which induce significant RG effects on the down-type Higgs mass to make it equal to the up-type Higgs mass in low-energy Higgs potential. Consequently the condition for positive M_A^2 naively seems difficult to be satisfied.

In the present model we now consider, the separation of two Higgs mass parameters is produced by y_τ and g_1 [6]. Notice that, if one took a limit y_τ, $g_1 \rightarrow 0$, the theory has an SU(2) symmetry which is identified to the global version of SU(2)$_R$ in the Pati-Salam unification group SU(4) × SU(2)$_L$ × SU(2)$_R$ [40]. In the symmetric limit, two Higgs SUSY-breaking masses are identical and the radiative EWSB does not occur. The positive value of c_M (gaugino mass effect) reflects the fact that the SU(2)$_R$ breaking (y_τ, $g_1 \neq 0$) induces $y_t > y_b$ in the RG evolution, and then lowers $m_{H_u}^2$ than $m_{H_d}^2$ at low energy, which difference is enhanced in the case of large gaugino mass. On the other hand, the negative value of c_{ms} (scalar mass effect) is a result that the SU(2)$_R$ breaking ($y_\tau \neq 0$ and the absence of neutrino Yukawa coupling) induces $m_{H_u}^2 > m_{H_d}^2$ at low energy, which is enhanced by larger scalar masses in the RG evolution. As a result, if smaller terms of A_0 and ξ are neglected,

the experimental lower bound on CP-odd neutral Higgs mass $M_A \gtrsim M_Z$ gives the following restriction between gaugino and scalar mass parameters:

$$M_{1/2}^2 \gtrsim \frac{-c_{ms}\tilde{m}_0^2 + 2M_Z^2}{c_M}. \tag{3.10}$$

It is found from explicit numerical values in figure 3 that the right-hand side is larger than $\tilde{m}_0^2$, and hence the above restriction is conservatively rewritten as

$$M_{1/2}^2 \gtrsim \tilde{m}_0^2. \tag{3.11}$$

This inequality is an important and strong constraint on the GUT-scale SUSY-breaking parameters; a half of parameter space is ruled out. It is also noticed that M_A is bounded from above by gaugino mass parameter

$$M_A^2 \lesssim c_M M_{1/2}^2 - M_Z^2. \tag{3.12}$$

In the SO(10)-type unification, therefore, the CP-odd neutral Higgs boson is generally predicted to be light [6]. The constraint (3.11) is not sensitive to the other parameters A_0 and ξ. The A_0 dependent terms have only tiny effects because the coefficients c_A and $c_{\rm AM}$ are very small, $|c_A|, |c_{\rm AM}| \lesssim \mathcal{O}(0.01)$. An extremely large value of $|A_0|$ tends to make the EWSB vacuum unstable [41] and is disfavored. As for the ξ dependence, the CP-odd neutral Higgs mass is a bit affected, depending on the sign of ξ. However ξ is constrained by other superparticle mass bounds; in particular, a large value of $|\xi|$ leads to scalar tau being the lightest supersymmetric particle (LSP), and hence the ξ term cannot be so large.

Let us turn to studying the second condition (3.9) concerning the higgsino masses. The numerical solution of the RG equations (d's in figure 3) indicates that the dominant positive contribution is the gaugino mass effect (d_M). The only possible correction comes from the scalar mass effect, especially the ξ term (d_{md}), which can lower $|\mu|^2$ when $\xi \gtrsim \frac{-d_{ms}}{d_{md}}$, e.g. $m_{10}^2 \gtrsim 1.3 m_{16}^2$ for $m_t^{\rm pole} = 178\,\text{GeV}$. The constraint (3.11) however means that such scalar mass contribution cannot be larger than that of gaugino, even if ξ has its maximal value 1. Furthermore a large value of ξ leads to the LSP scalar tau lepton and is disfavored. The other effects from the A_0 and M_Z terms are negligibly small. Thus the gaugino mass effect becomes dominant in large region of SUSY-breaking parameters; the low-energy $|\mu|$ is approximately given by the gaugino mass

$$|\mu|^2 \sim d_M M_{1/2}^2. \tag{3.13}$$

In this case, the lighter neutralinos and chargino become gaugino-like, since the unified gaugino mass at the GUT scale leads to $M_1 \simeq 0.4M_{1/2}$ and $M_2 \simeq 0.8M_{1/2}$ at the electroweak scale, which are generally smaller than $|\mu|$.

In these ways the radiative EWSB in the SO(10)-type unification requires a restricted type of low-energy superparticle spectrum. This is mainly due to the constraint (3.11) on the GUT-scale parameters which generally predicts (i) scalar quark masses are correlated with the gluino mass through the RG evolution and cannot be much larger than it, (ii) scalar leptons also cannot be much heavier than the SU(2)$_L$ gaugino, (iii) the gaugino

components are dominant in the lighter neutralinos and chargino, and (iv) a relatively light CP-odd neutral Higgs boson is expected. It is stressed here that eq. (3.13) and mass eigenvalues related to the μ parameter depend on the magnitude of ξ term in the RG solution (3.9), which is restricted by the inequality (3.11) and the requirement that the LSP is charge neutral. The relevance of ξ-dependent contribution will be investigated in later sections and found to sometimes play an important role in establishing the successful radiative EWSB in other scenarios.

In the next subsection a detailed study will be given for the experimental constraints in the minimal SO(10)-type scenario. Before proceeding to numerical analysis, we here present a summary of the results, referring to the mass spectrum naively expected from the above discussion. First, the threshold correction to bottom quark mass is generally not suppressed. That depends on the relative strength of scalar quark mass, gluino mass, and μ parameters. The EWSB constraints (3.11) and (3.13) mean that the PQ and R symmetries are largely violated, and the threshold correction becomes large as discussed in section 2.1. Secondly, the $b \to s\gamma$ amplitude becomes large since the charged Higgs contribution A_{H^+} is enhanced by a light charged Higgs boson $M_{H^+}^2 = M_A^2 + M_W^2$. In particular, for $\mu < 0$, the observation of $b \to s\gamma$ rare decay severely restricts the parameter space of the model. The enhancements of the threshold correction Δ_b and the $b \to s\gamma$ amplitude generally make the minimal SO(10)-type unification difficult to be consistent with the observation.

3.2 Parameter space analysis

We perform the numerical analysis of parameter space in the minimal SO(10)-type unification which space is allowed by the experimental constraints from the bottom quark mass and the $b \to s\gamma$ decay rate. The input parameters are the same as those in section 2, and the top quark mass is $m_t^{\text{pole}} = 178 \, \text{GeV}$. The unified Yukawa coupling y_G is evaluated by using the two-loop MSSM RG equations and one-loop SUSY threshold corrections to top and tau Yukawa couplings, Δ_t and Δ_τ, which are controlled by SUSY-breaking mass parameters. The low-energy threshold corrections to gauge couplings are also taken into account. Once y_G is determined, one can solve the EWSB conditions, the mass bound of CP-odd neutral Higgs boson, and the requirement of neutral LSP at the scale $Q = M_{\text{SUSY}}$ which is typically defined by scalar quark mass parameters as $M_{\text{SUSY}} = (m_{\tilde{Q}33}^2 m_{\tilde{u}33}^2)^{1/4}$. The current experimental lower bounds on gaugino masses are included as in the previous section, and the mass bounds on the scalar top, bottom, tau are $m_{\tilde{t}_1} > 95.7 \, \text{GeV}$, $m_{\tilde{b}_1} > 89 \, \text{GeV}$, and $m_{\tilde{\tau}_1} > 81.9 \, \text{GeV}$ (95% CL), respectively [27]. Finally the bottom quark mass is estimated with one-loop SUSY threshold correction and two-loop SM QCD correction [29, 30, 5], and the $b \to s\gamma$ branching ratio is calculated according to the formulas in [36].

Figure 4 shows that the parameter space consistent with the radiative EWSB, the experimental mass bounds of superparticles, and the requirement of neutral LSP. In the figures, the predictions of bottom quark mass $m_b^{\overline{\text{MS}}}(m_b)$ and the $b \to s\gamma$ branching ratio are shown in the allowed parameter regions. For simplicity, vanishing scalar trilinear couplings and the universal scalar masses ($A_0 = \xi = 0$) have been assumed in the figures. The numerical result here shows that the GUT-scale gaugino mass $M_{1/2}$ must be larger than
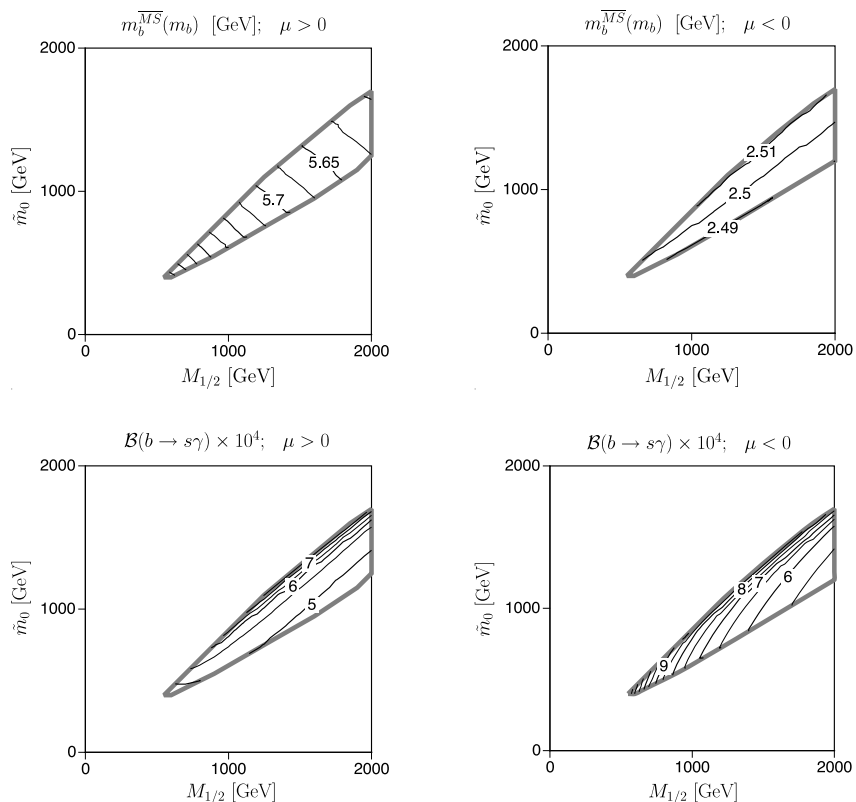
Figure 4: The parameter space consistent with the radiative EWSB, the experimental mass bounds of superparticles, and the requirement of neutral LSP in the minimal SO(10)-type unification. The bottom quark mass $m_b^{\overline{\rm MS}}(m_b)$ and the $b \to s\gamma$ branching ratio are also shown in the figures. The GUT-scale scalar mass parameters are set as $A_0 = \xi = 0$. The two-loop MSSM RG equations for gauge and Yukawa couplings and the one-loop ones for dimensionful parameters are used. The one-loop SUSY threshold corrections to gauge and Yukawa couplings are included. The radiative EWSB conditions are solved by use of the one-loop effective potential at the scale $Q = M_{\rm SUSY}$ which is defined by the scalar quark masses as $M_{\rm SUSY} = (m_{\tilde{Q}33}^2 m_{\tilde{u}33}^2)^{1/4}$. In each figure, the left-top region is excluded by the mass bound of CP-odd neutral Higgs boson and the right-bottom region is ruled out from the fact that the scalar tau lepton becomes the LSP.

the universal scalar mass $\tilde{m}_0$, which confirms the previous analysis (3.11). Too a large value of gaugino mass $M_{1/2} > 2\tilde{m}_0$ is excluded by the requirement of neutral LSP, in which region the scalar tau lepton becomes the LSP. The unification of Yukawa couplings generally makes scalar lepton mass eigenvalues small due to the y_τ contribution to scalar lepton masses in the RG evolution and large left-right mixing elements in the scalar tau mass matrix which is proportional to $\tan\beta$. In the region allowed by the experimental constraints, SUSY spectrum is severely constrained from the inequality (3.11). For example, in the present model, the μ parameter at low energy has a strong correlation with the gaugino mass; $|\mu|^2 \simeq 1.4 M_{1/2}^2$. Therefore the lightest neutralino and chargino become gaugino-like. Also scalar quark masses are correlated with $M_{1/2}$ and have few dependence on the initial value $\tilde{m}_0$; the mass of light scalar top is given by $m_{\tilde{t}_1}^2 \simeq 3.1 M_{1/2}^2$ or equivalently $m_{\tilde{t}_1}^2 \simeq 0.5 M_3^2$.

The CP-odd neutral Higgs boson is generally expected to be light in the minimal SO(10)-type unification; $M_A^2 \lesssim 0.06 M_{1/2}^2$ in the parameter region in figure 4.

The minimal SO(10)-type unification leads to the constrained mass spectrum largely violating the PQ and R symmetries. That predicts a large value of the threshold correction $|\Delta_b|$, that is, $0.3 \lesssim |\Delta_b| \lesssim 0.4$ in the allowed parameter region of figure 4. As examined in section 2.1, the Yukawa unification requires a small value of $|\Delta_b|$ in order to obtain the bottom quark mass within the experimentally allowed range. In the present case, too large a magnitude of the finite threshold correction $|\Delta_b|$ spoils the successful bottom mass prediction. It is observed from figure 4 that m_b is too large for $\mu > 0$ and too small for $\mu < 0$. The bottom mass prediction has little sensitivity to the overall SUSY-breaking scale. Further the $b \to s\gamma$ branching ratio generally becomes large in the minimal SO(10)-type unification. This is due to the large amplitude $|A_{H^+}|$ enhanced by a small value of the charged Higgs boson mass. As discussed before, a cancellation between the amplitudes A_{H^+} and $A_{\tilde{\chi}^+}$ is possible with a positive μ parameter. Now we have a hierarchical spectrum that scalar quarks are much heavier than the charged Higgs boson, and then $|A_{H^+}|$ becomes large; $|A_{H^+}| \gtrsim 1.3 |A_{\tilde{\chi}^+}|$ in the parameter region of figure 4. Therefore the cancellation among the amplitudes is not enough to suppress the non-SM contributions to $b \to s\gamma$ transition even for $\mu > 0$, and the experimental constraint is serious. For the $\mu < 0$ case, the constraint becomes severer than the $\mu > 0$ case.

Next, let us examine the parameter dependences on A_0 and ξ, that is, non-vanishing scalar trilinear couplings and the difference of SUSY-breaking masses between matter and Higgs fields. Figures 5 and 6 are the same as figure 4 but for $M_{1/2}$–ξ and A_0–ξ parameter spaces, respectively. The experimental mass bound on CP-odd neutral Higgs boson excludes the left (right) side of parameter space in figure 5 (figure 6). The charged LSP regions correspond to the right-top (left and top) region in figure 5 (figure 6).

The CP-odd neutral Higgs mass has little dependence on ξ. This behavior can be understood from the tiny value of c_{md} in (3.8). As mentioned before, the CP-odd neutral Higgs mass is scaled with the difference between the Higgs mass parameters. A non-zero ξ generates a separation between Higgs and matter scalar masses at the GUT scale, but does not directly contribute to the separation inside Higgs masses. Thus the experimental bound on CP-odd neutral Higgs mass is not relaxed with the freedom of ξ. The ξ dependence of the $b \to s\gamma$ branching ratio is also small. This is a consequence of small ξ dependence on the charged Higgs mass. In contrast to the CP-odd neutral Higgs mass, the scalar tau mass has larger dependence on ξ; a larger negative ξ raises the mass of scalar tau and the constraint from the LSP is relaxed. The more ξ increases, the lighter scalar tau lepton is. The roughly upper half of the $M_{1/2}$–ξ plane is excluded by the LSP condition. This in turn implies that the ξ-dependent term (d_{md}) in the RG solution (3.9) is much smaller than the gaugino mass effect (d_M), and $|\mu|$ cannot be smaller than $M_{1/2}$; for example, $1.2 \lesssim |\mu|^2/M_{1/2}^2 \lesssim 2.0$ in the allowed parameter region in figure 5. For a negative ξ, the μ parameter is increased which enhances the magnitude of threshold correction Δ_b, and consequently, the bottom quark mass becomes large.

The A_0 dependence of CP-odd neutral Higgs mass is more relevant than that on ξ. It is found from the RG solution (3.9) that, for a positive (negative) A_0, the CP-odd neutral
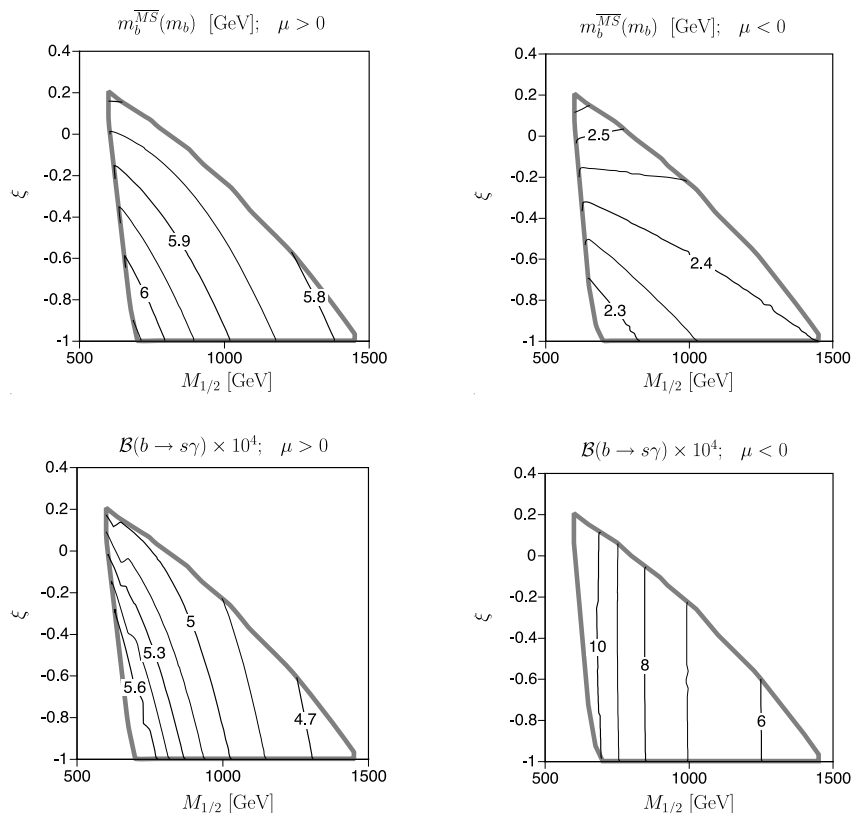
Figure 5: The same as figure 4 but for $M_{1/2}$–ξ parameter space. The GUT-scale scalar mass parameters are set as $\tilde{m}_0 = 500\,\text{GeV}$ and $A_0 = 0$. In each figure, the narrow left side is excluded by the experimental mass bound on CP-odd neutral Higgs boson and the right-top region is ruled out from the fact that the scalar tau lepton becomes the LSP.

Higgs mass generally becomes smaller (larger) than the $A_0 = 0$ case. In figure 6, we have $90\,\text{GeV} \lesssim M_A \lesssim 180\,\text{GeV}$. Note that a rather large value of $|A_0|$ lowers M_A even for a negative value of A_0 because such a large $|A_0|$ enhances the tau Yukawa effect in the RG evolution which lowers $m_{H_d}^2$ rather than $m_{H_u}^2$ in low-energy regime. In a similar way, the masses of scalar tau lepton and scalar quarks have sizable dependences on A_0. A large $|A_0|$ lowers these masses through the RG evolution down to low energy, and in particular, the large $|A_0|$ region is excluded since the LSP is scalar tau lepton. As seen from (3.9), a negative A_0 raises $|\mu|$ and then the bottom mass prediction is enhanced and becomes worse. Contrary to this behavior, the $b \to s\gamma$ branching ratio is decreased by a negative A_0. This is a consequence of the A_0 dependence of M_{H^+}; a negative A_0 raises M_{H^+} and lowers A_{H^+} whose absolute value is generally larger than $A_{\tilde{\chi}^+}$ in the minimal SO(10)-type unification. Therefore the experimental constraint from the $b \to s\gamma$ process is relaxed.

These ξ and A_0 dependences slightly modify the $M_{1/2}$–$\tilde{m}_0$ parameter space which is consistent with the mass bound of CP-odd neutral Higgs boson and the requirement of neutral LSP (figure 4). However such change is small and does not allow the EWSB with $M_{1/2}, |\mu| \ll \tilde{m}_0$, with which spectrum the threshold correction to bottom quark mass is
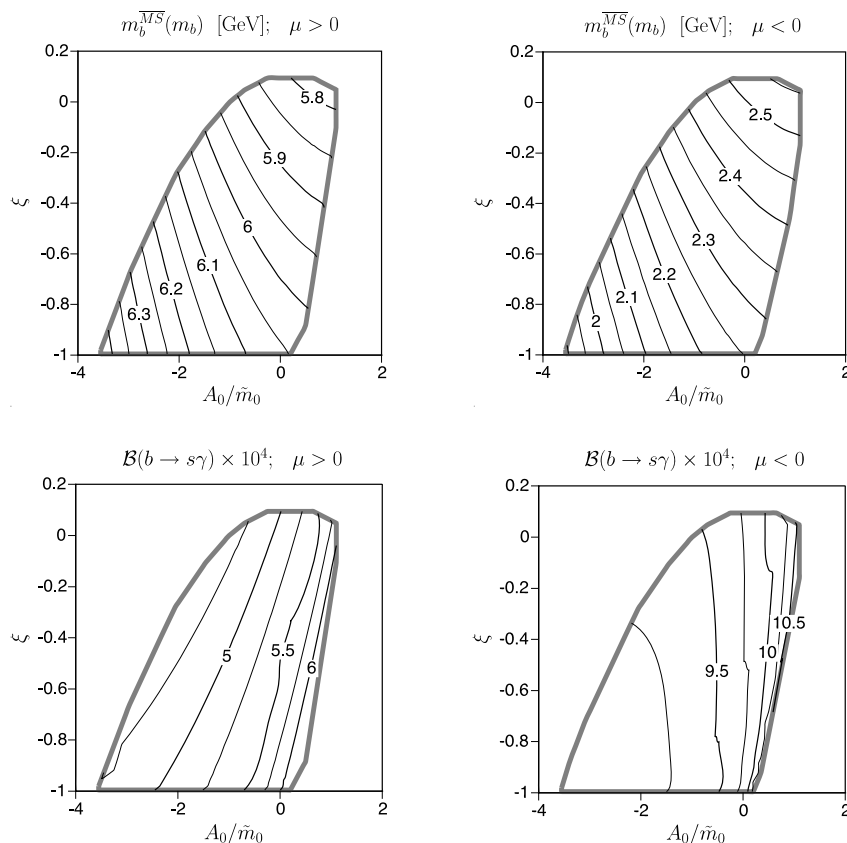
Figure 6: The same as figure 4 but for A_0–ξ parameter space. The GUT-scale scalar mass parameters are set as $\tilde{m}_0 = 500\,\mathrm{GeV}$ and $M_{1/2} = 700\,\mathrm{GeV}$. In each figure, the right side is excluded by the experimental mass bound on CP-odd neutral Higgs boson and the left and top regions are ruled out from the fact that the scalar tau lepton becomes the LSP.

suppressed in a technically natural way. The minimal SO(10)-type unification is therefore difficult to reproduce the observed value of bottom quark mass. The $b \to s\gamma$ rare decay process also restricts a light superparticle spectrum at low-energy regime.

3.3 Discussions

As shown in section 2.1, the enhanced threshold corrections to bottom quark mass are proportion to μM_3 and μA_t, which are controlled by the PQ and R symmetries. These symmetries are useful in the SO(10)-type unification since the suppression of the threshold correction is required to attain the experimentally allowed fermion masses in the Yukawa unification [5, 7, 11]. It is however found that the SO(10)-type unification does not allow to impose the PQ and R symmetries because a successful EWSB, in particular, the separation of two Higgs scalar masses, requires a symmetry-violating condition $M_{1/2} \gtrsim \tilde{m}_0$ for the GUT-scale SUSY-breaking parameters. This behavior is a consequence that the RG solution (3.8) receives a sizable negative contribution from scalar masses. If the coefficient c_{ms} in (3.8) turns to be positive or there exist some additional positive contributions, the

radiative EWSB with a small gaugino mass $M_{1/2} < \tilde{m}_0$ is possible, and accordingly one could take $M_{1/2}$, A_0, $B_0 \ll \tilde{m}_0$, that is, the R symmetric radiative EWSB is viable. Moreover, in such a case, the PQ symmetry is also realized since the gaugino mass effect in the solution (3.9) could be canceled by the scalar mass contribution.

It is pointed out [42] that the D-term effect of additional U(1) symmetry contributes to the CP-odd neutral Higgs mass without disturbing the PQ and R symmetries. Then the radiative EWSB with $M_{1/2} < \tilde{m}_0$ is available with an appropriate D-term contribution included. Along this line, there are various studies about phenomenological aspects in SO(10) models (see, e.g. [7, 10, 9]). On the other hand, a similar type of EWSB is also viable with some specific types of non-universal scalar masses which do not respect SO(10) unified gauge symmetry. In these cases, approximate PQ and R symmetries are realized in low-energy mass spectrum.

In the following sections, we investigate alternative possibilities to attain PQ and/or R symmetric spectrum which include the effects inspired by neutrino physics in SO(10) models, and will find new types of radiative EWSB scenarios.

4. SO(10) unification with neutrino couplings

It has been indicated by various recent experiments that neutrinos have tiny mass scale less than a few eV, which is extremely smaller than the other SM fermion masses. In the framework of SO(10) unification, a 16-plet contains a single field under the SM gauge group which may be naturally identified to a right-handed neutrino. The SO(10)-invariant superpotential term $16\,16\,10_H$ generates neutrino Yukawa couplings among the left- and right-handed neutrinos and the up-type Higgs boson. Thus neutrinos obtain a similar size of Dirac masses to the other SM fermions and the observed tiny mass scale seems unnatural. A promising way to cure this problem is to introduce large Majorana masses for right-handed neutrinos. Integrated out the heavy right-handed neutrinos, tiny Majorana masses are generated for left-handed neutrinos [13]. Thus the superpotential terms below the GUT scale in this scenario is given by

$$W = W_{\text{MSSM}} + L_i(Y_\nu)_{ij}\bar{\nu}_j H_u + \frac{1}{2}\bar{\nu}_i(\mathcal{M}_\nu)_{ij}\bar{\nu}_j. \tag{4.1}$$

The last two terms are introduced in addition to the MSSM superpotential (2.1) where Y_ν is the neutrino Yukawa matrix and $\mathcal{M}_\nu$ denotes large-scale Majorana masses for right-handed neutrino superfields $\bar{\nu}_i$ ($i = 1, 2, 3$). As in the previous Yukawa unified scenarios, we naturally have a hierarchical order of neutrino Yukawa couplings and then only the third diagonal element is large; $(Y_\nu)_{33} \equiv y_\nu \sim \mathcal{O}(1)$, which is expected to be of the same order of the top Yukawa coupling in SO(10) unification. Such a hierarchy assumption might also be applied to the Majorana mass matrix of right-handed neutrinos. In the following analysis we simply have the 3-3 element of the Majorana mass matrix as $(\mathcal{M}_\nu)_{33} \equiv M_\nu = 10^{14}$ GeV. The minimal SO(10)-type unification discussed in the previous section corresponds to the case that M_ν is equal to or larger than the GUT-breaking scale. The other matrix elements of neutrino couplings are smaller than these dominant 3-3 elements, and might be responsible for explaining the observed large generation mixing of light neutrinos [43]. The details

of these small matrix elements are completely irrelevant to the following RG analysis and can be dropped. If there were other Yukawa matrix elements than $(Y_\nu)_{33}$ which take $\mathcal{O}(1)$ values, the effects of neutrino couplings are enhanced accordingly. However in this paper we assume a conservative case that only the tau-neutrino Yukawa coupling is large.

In this section, we study the unification scenario with $M_\nu < M_G$. As a result, the effects of neutrino couplings become important in the RG evolution between M_G and M_ν. The influences of neutrino Yukawa couplings have been studied concerning on the gauge coupling unification [23] and the third-generation fermion masses [21, 22]. Also in ref. [44], the radiative EWSB is examined in the case of large violation of PQ and R symmetries. It was found that such neutrino Yukawa effect is less than a few percents and the qualitative discussion, e.g. that given in section 2.1, is also applied to our present case. In particular, a successful prediction of bottom quark mass still requires that the threshold correction Δ_b at SUSY-breaking scale must be suppressed. On the other hand, as will be shown below, neutrino couplings play a significant role in the RG evolution of SUSY-breaking parameters. Thus the picture of radiative EWSB and superparticle mass spectrum are found to be considerably altered from the minimal SO(10)-type unification. We will show that neutrino coupling effects make it possible to attain PQ and R symmetric radiative EWSB which is preferred by the bottom quark mass prediction in the large $\tan\beta$ case.

4.1 Radiative EWSB with large neutrino couplings

Let us first see how the radiative EWSB scenario is altered by introducing neutrino couplings. The right-handed neutrinos contribute the one-loop RG equations for Higgs SUSY-breaking mass parameters which are given by

$$\frac{dm_{H_u}^2}{d\ln Q} = \left.\frac{dm_{H_u}^2}{d\ln Q}\right|_{\text{MSSM}} + \frac{y_\nu^2}{8\pi^2}\big(m_{H_u}^2 + m_{\tilde{L}}^2 + m_{\tilde{\nu}}^2 + A_\nu^2\big), \tag{4.2}$$

$$\frac{dm_{H_d}^2}{d\ln Q} = \left.\frac{dm_{H_d}^2}{d\ln Q}\right|_{\text{MSSM}} \tag{4.3}$$

above the decoupling scale of (the third-generation) right-handed neutrino M_ν. The mass parameters $m_{\tilde{L}}^2$, $m_{\tilde{\nu}}^2$, and A_ν are the abbreviations of the third diagonal elements of left-, right-handed scalar neutrino mass matrices, and trilinear coupling of scalar neutrinos, respectively. The wavefunction renormalization of up-type Higgs scalar is affected by propagating right-handed neutrinos but that of down-type Higgs scalar is not. As a result, only the RG equation of up-type Higgs mass (4.2) receives the additional terms ($\propto y_\nu^2$) from the neutrino sector. These terms are naturally positive in the RG evolution down to M_ν and lowers $m_{H_u}^2$ in the infrared region, compared to the MSSM prediction. The down-type Higgs mass is not altered at one-loop order.

As seen in the previous section, the SO(10)-type Yukawa unification generally leads to only a tiny difference between up- and down-type Higgs masses at the electroweak scale and excludes a large portion of parameter space by the experimental bound of CP-odd neutral Higgs mass which is proportional to that mass difference. This fact reflects the $SU(2)_R$ symmetry which is violated only by the small $U(1)_Y$ gauge coupling and the

absence of right-handed neutrinos. In particular, the latter decreases the down-type Higgs mass by tau Yukawa effect in the RG evolution. In the present scenario, including the neutrino couplings in the RG equation of $m_{H_u}^2$ tends to cancel the tau Yukawa effect and provides a positive contribution to the CP-odd neutral Higgs mass squared. Consequently the radiative EWSB is expected to be made more natural than the minimal SO(10)-type unification.

The one-loop RG equations for scalar lepton masses are also affected by the neutrino couplings. In particular the scalar tau mass eigenvalue is decreased and the requirement of charge-neutral LSP becomes severer than the minimal SO(10)-type unification. If considered the R symmetric radiative EWSB, the scalar tau lepton could readily be made heavier than the lightest neutralino. That will be checked in the numerical analysis below.

In the following we simply take $y_\nu = y_G$ at the GUT scale for comparison to the analysis of Yukawa unification in the previous sections. Evaluating the RG evolution of Higgs mass parameters, we obtain the RG solutions in the EWSB vacuum:

$$
\begin{aligned}
M_A^2 &= (e_{ms} + e_{md}\xi)\tilde{m}_0^2 + e_{m\nu}m_{\nu 0}^2 + e_M M_{1/2}^2 + e_{AM}A_0 M_{1/2} + e_{A_\nu M}A_{\nu 0}M_{1/2} \\
&\quad + e_A A_0^2 + e_{A_\nu}A_{\nu 0}^2 + e_{AA_\nu}A_0 A_{\nu 0} - M_Z^2,
\end{aligned} \tag{4.4}
$$

$$
\begin{aligned}
|\mu|^2 &= (f_{ms} + f_{md}\xi)\tilde{m}_0^2 + f_{m\nu}m_{\nu 0}^2 + f_M M_{1/2}^2 + f_{AM}A_0 M_{1/2} + f_{A_\nu M}A_{\nu 0}M_{1/2} \\
&\quad + f_A A_0^2 + f_{A_\nu}A_{\nu 0}^2 + f_{AA_\nu}A_0 A_{\nu 0} - \frac{M_Z^2}{2},
\end{aligned} \tag{4.5}
$$

where $m_{\nu 0}^2$ and $A_{\nu 0}$ are the boundary values of m_ν^2 and A_ν at the GUT-breaking scale. In order to clarify the effects of neutrino couplings, we have separated them from the other parameters of charged fields, the latter of which are simply assumed to be unified at the GUT scale for comparison to the previous SO(10)-type unified scenario. The separation of neutrino SUSY-breaking parameters while keeping (approximate) Yukawa unification is dynamically corroborated, e.g. in the framework that right-handed neutrinos contain low-energy remnants of gauge-singlet superfields. When the mixture is tiny between such extra singlets and the third-generation 1_3 in the 16_3 multiplet, the Yukawa unification is almost preserved. On the other hand, SUSY-breaking parameters of neutrinos are significantly modified if the extra singlets receive larger breaking effects than the ordinary matter. That could be easily realized if the extra fields directly couple to SUSY-breaking sector and the others are not. A simpler and alternative mechanism for separating the neutrino effect is to assume that neutrino Yukawa couplings for the first and second generations have $\mathcal{O}(1)$ values, which turn out to enhance RG running effects of neutrino couplings. That is possible since the neutrino mass spectrum has not been experimentally determined unlike the other charged fermions. Anyway the neutrino couplings are regarded as free parameters and can be large. It may be interesting to note that large values of neutrino couplings do not cause phenomenological problems such as the destabilization of the EWSB vacuum because of the existence of huge supersymmetric Majorana masses of right-handed neutrinos.

The coefficients in the solutions (4.4) and (4.5) are roughly estimated from the RG equation (4.2) of the up-type Higgs mass parameter. The RG running between M_G and M_ν generates a departure $\delta m_{H_u}^2$ from the MSSM prediction of up-type Higgs mass. At
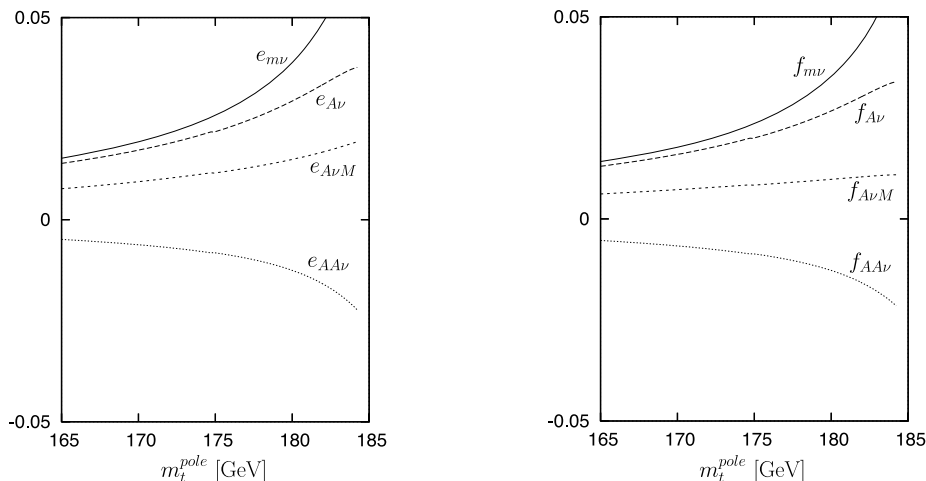
Figure 7: Typical behaviors of the coefficients e's and f's in the RG solutions (4.4) and (4.5) evaluated at the renormalization scale $Q = 1\,\text{TeV}$. Here we take input parameters the same as in the previous figure 3 in addition to the decoupling scale of the third-generation right-handed neutrino $M_\nu = 10^{14}\,\text{GeV}$.

one-loop order, the neutrino coupling effect is given by

$$\delta m_{H_u}^2 \;=\; -(2\tilde{m}_0^2 + m_{\tilde\nu 0}^2 + A_{\nu 0}^2)\,\epsilon + \mathcal{O}(\epsilon^2), \tag{4.6}$$

where the positive parameter ϵ is

$$\epsilon \;=\; \frac{y_\nu^2}{8\pi^2}\ln\left(\frac{M_G}{M_\nu}\right) \;\simeq\; 0.06\,y_\nu^2. \tag{4.7}$$

We find from this expression that the coefficients satisfy $e_{ms} \simeq c_{ms} + 2\epsilon$ and $e_{m\nu} \simeq e_{A_\nu} \simeq \epsilon$. The others, e_{md}, e_M, e_A and e_{AM}, are almost the same as the corresponding coefficients c's in the MSSM case, and also the neutrino part $e_{A_\nu M}$ and e_{AA_ν} are the next-to-leading order of ϵ. The similar behaviors are expected to hold for the solution (4.5) since the leading-order neutrino RG effect comes from the modification of $m_{H_u}^2$ in low-energy regime. We checked these behaviors by numerically solving the RG equations. Figure 7 represents typical values of e's and f's as the functions of the top quark mass, especially concerning the neutrino parameters $m_{\tilde\nu 0}^2$ and $A_{\nu 0}$. In the calculation, the input parameters are taken as the same as in figure 3, and the EWSB conditions are solved at the renormalization scale $Q = 1\,\text{TeV}$. It is found from the figures that the above analytic estimation is consistent with the exact numerical one. Thus the neutrino coupling effects are characterized by ϵ.

While ϵ is not so large, it provides significant effect on the radiative EWSB. An important point here is that, in addition to the usual gaugino mass effect, there are new sources of positive contribution to the CP-odd neutral Higgs mass squared M_A^2, that is, the neutrino effects $e_{m\nu} m_{\tilde\nu 0}^2 + e_{A_\nu} A_{\nu 0}^2$ in (4.4). These terms can raise M_A without leading to a large gaugino mass, and the experimental bound on M_A is made consistent with the R symmetric low-energy superparticle spectrum which allows the prediction of bottom quark

mass well within the experimental range. In the limit $M_{1/2}, A_0 \to 0$, the RG solution (4.4) becomes

$$M_A^2 = \left[e_{ms} + e_{md}\xi + (e_{m\nu}N^2 + e_{A\nu}N_A^2)(1 - \xi) \right] \tilde{m}_0^2 - M_Z^2, \tag{4.8}$$

where N and N_A represent the effects of neutrino couplings compared to other matter SUSY-breaking parameters; $N^2 = m_{\tilde{\nu}0}^2/m_{16}^2$ and $N_A^2 = A_{\nu 0}^2/m_{16}^2$. If one neglects M_Z and ξ, the positive M_A^2 implies

$$N^2 + N_A^2 \; \gtrsim \; \frac{-e_{ms}}{\epsilon} \; \sim \; 4. \tag{4.9}$$

Therefore only a few times larger values of neutrino couplings are needed to obtain phenomenologically preferred mass spectrum. A non-vanishing ξ has little dependence on this lower bound as will be shown in the following numerical analysis.

The PQ symmetric spectrum is also available if one considers the above R symmetric radiative EWSB and a suitable value of ξ parameter. In the solution for $|\mu|^2$ (4.5), the positive contribution from gaugino masses can be cancelled by the ξ term (f_{md}). Note that there also exists the neutrino coupling effect in the evaluation of μ parameter which raises $|\mu|$ compared with the minimal SO(10)-type unification. However this effect is not important since the characteristic size of neutrino coupling effect (ϵ) is much smaller than the dominant contributions from scalar masses (f_{md}) and gaugino masses (f_M).

The neutrino coupling effects in the RG evolution of mass parameters lead to low-energy superparticle spectrum quite different from that in the minimal SO(10)-type unification. In particular, the PQ and R symmetries are found to appear in the mass spectrum by introducing natural, sizable effects of neutrino couplings. Such a spectrum is known to be favorable to low-energy phenomenology in the large $\tan\beta$ case. In the next subsection, we will examine the parameter space consistent with the PQ and R symmetric radiative EWSB, the experimental mass bounds of superparticles, and the requirement of neutral LSP and also discuss the predictions of bottom quark mass and the $b \to s\gamma$ rare decay.

4.2 Parameter space analysis

Let us perform the parameter space analysis of the SO(10) unification with neutrino couplings. We particularly focus on the realization of the PQ and/or R symmetric radiative EWSB and its low-energy phenomenology. This type of radiative EWSB is triggered by moderate values of mass and/or trilinear coupling of the third-generation right-handed neutrino [see eq. (4.9)]. In the RG evolution, scalar masses and trilinear couplings generally have similar effects since they appear together in beta functions. A difference might be generated in the evolution of neutrino trilinear coupling between M_G and M_ν, which results in, e.g. the difference between $e_{m\nu}$ and e_{A_ν} in the solution (4.4). Such differences are however generally small, and the two cases with a large N and with a large N_A lead to almost the same low-energy superparticle spectrum. In what follows we will show, as an illustration, the parameter space analysis for the case of varying scalar neutrino mass (N) with the vanishing neutrino trilinear couplings ($N_A = 0$).

The two important model parameters for successful radiative EWSB are the scalar neutrino mass denoted by N and the matter/Higgs mass discrepancy parameter ξ defined in (3.6). Figure 8 shows the parameter regions of (N, ξ) consistent with the radiative
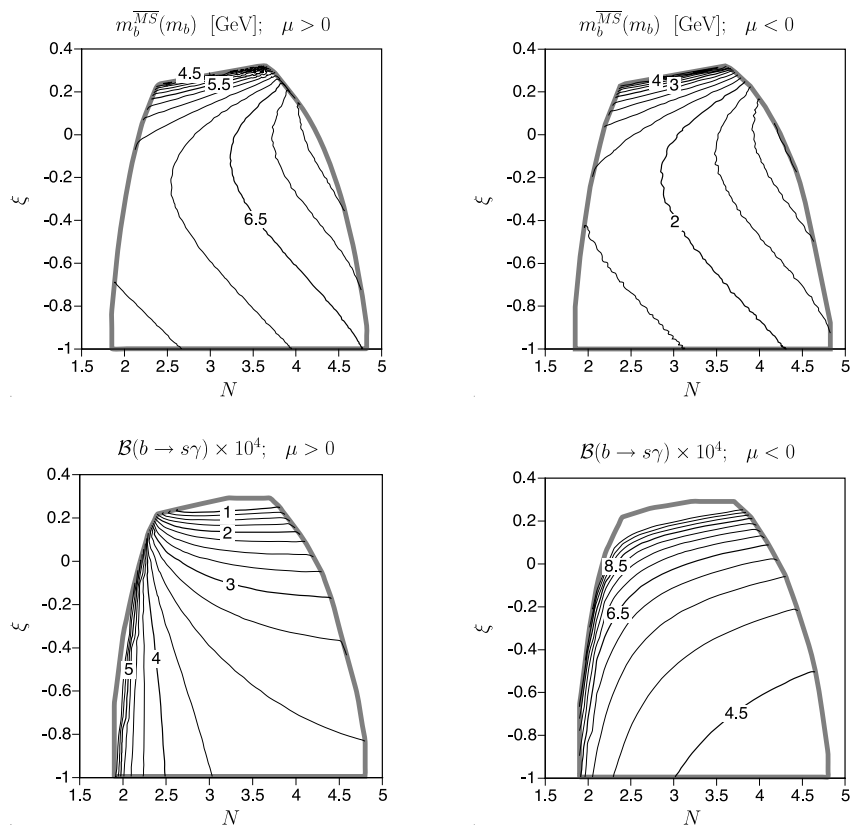
Figure 8: The parameter space consistent with the radiative EWSB, the experimental mass bounds of superparticles, and the requirement of neutral LSP in the SO(10) unification with neutrino couplings. The horizontal and vertical axes denote the SUSY-breaking mass of third-generation scalar neutrino and the matter/Higgs discrepancy parameter, respectively. The bottom quark mass $m_b^{\overline{MS}}(m_b)$ and the $b \to s\gamma$ branching ratio are also shown in the figures. The GUT-scale mass parameters are set as $M_{1/2} = 300\,\text{GeV}$, $\tilde{m}_0 = 2\,\text{TeV}$, and $A_0 = 0$. The other input parameters are taken as the same as in the previous figure 4. The radiative EWSB conditions, the superparticle mass bounds, the SUSY threshold corrections and the $b \to s\gamma$ branching ratio are calculated in the same way as figure 4. In each figure, the left, top, and right regions are excluded, respectively, by the mass bound of CP-odd neutral Higgs boson, the lower bounds of chargino and neutralino masses, and the requirement that the scalar tau lepton should not be the LSP.

EWSB conditions, the experimental bounds on superparticle masses, and the requirement of neutral LSP. In the figures, the predictions of bottom quark mass $m_b^{\overline{MS}}(m_b)$ and the $b \to s\gamma$ branching ratio are shown in the allowed parameter regions. The boundary values of SUSY-breaking variables at the GUT scale are $M_{1/2} = 300\,\text{GeV}$, $\tilde{m}_0 = 2\,\text{TeV}$, and $A_0 = 0$. The other input parameters are taken as the same as in the previous figure 4. In figure 8, the left side of the parameter space is excluded by the experimental mass bound of CP-odd neutral Higgs boson. This suggests that a large SUSY-breaking mass of right-handed scalar neutrino raises the Higgs mass. It is also found that the analytic estimation (4.9) is well satisfied and its ξ dependence is small. On the other hand, the ξ

dependence of μ parameter is relatively large: a positive, larger value of ξ rapidly lowers the prediction of $|\mu|$ for $M_{1/2} \ll \tilde{m}_0$. That rules out the top region in each figure by the lower mass bounds of charginos and neutralinos. In the parameter space of figure 8, we obtain the CP-odd neutral Higgs mass $M_A \lesssim 3M_{1/2}$ and the μ parameter as small as $50\,\mathrm{GeV}$. Too large values of ξ and right-handed scalar neutrino mass reduce the mass eigenvalues of scalar lepton doublets through the RG evolution down to low-energy regime. This is encoded to the right-top region excluded by the cosmological requirement that charged field (right-handed scalar tau) should not be the LSP.

One of the notable features of the present scenario is that there exists the parameter region (the top region in the figures) excluded by the mass bounds of charginos and neutralinos. In other words, a relatively small value of μ parameter is consistent with the radiative EWSB. That is achieved with the ξ-dependent negative contribution to $|\mu|^2$ in the solution (4.5). In this parameter region, the gaugino masses and holomorphic couplings A, B and μ can be much smaller than non-holomorphic scalar masses, and thus an approximate PQ and R symmetric spectrum is realized. Such a hierarchical mass pattern was incompatible with the minimal SO(10)-type unification in the previous section. These symmetries tend to suppress the low-energy threshold correction to the bottom quark mass. We find that, for both signs of the μ parameter, successful predictions of bottom quark mass are obtained on the top margin of the allowed parameter region in figure 8 where μ takes a relatively small value, and the lighter chargino is gaugino-like and becomes lighter than about $120\,\mathrm{GeV}$.

The $b \to s\gamma$ branching ratio is decreased as the right-handed scalar neutrino mass N, since the charged Higgs mediated amplitude A_{H^+} is suppressed by the CP-odd neutral Higgs mass. The branching ratio also has the ξ dependence since a negative large value of ξ increases scalar quark masses. In the PQ and R symmetric region, where a strong suppression of the threshold correction Δ_b is obtained, the $b \to s\gamma$ decay constraint apparently seems severe even for the positive μ case. This is because the chargino contribution $A_{\tilde{\chi}^+}$ is enhanced to be larger than A_{H^+} by small mass eigenvalues of charginos. As a result, large scalar masses are favored for both signs of μ in order to avoid the experimental constraint from $b \to s\gamma$ rare decay.

The above discussion implies that the PQ and R symmetric superparticle spectrum leads to phenomenologically preferred values of the bottom quark mass and the $b \to s\gamma$ branching ratio if the matter scalars become heavy. That can be seen in figure 9 which is the same as the previous figure 8 but for the $\tilde{m}_0$–ξ parameter space. In general, the left side region is excluded by the LSP scalar tau lepton and the top one by the experimental mass bounds of CP-odd neutral Higgs boson and chargino.

For the $\mu > 0$ case, there is the parameter region consistent with the experimental ranges of bottom quark mass and $b \to s\gamma$ decay rate. In the allowed parameter region, the universal scalar mass is found to become $\tilde{m}_0 \gtrsim 2.5\,\mathrm{TeV}$. The universal gaugino mass is much smaller than scalar masses as $M_{1/2} \lesssim 0.12\tilde{m}_0$, and μ is also suppressed as $\mu \lesssim 0.15\tilde{m}_0$ at the electroweak scale. Thus the approximate PQ and R symmetries are easily realized in the spectrum, which suppresses SUSY threshold corrections and reproduces the observed value of bottom quark mass. For $\tilde{m}_0 \lesssim 4\,\mathrm{TeV}$, relatively a small μ parameter is required to
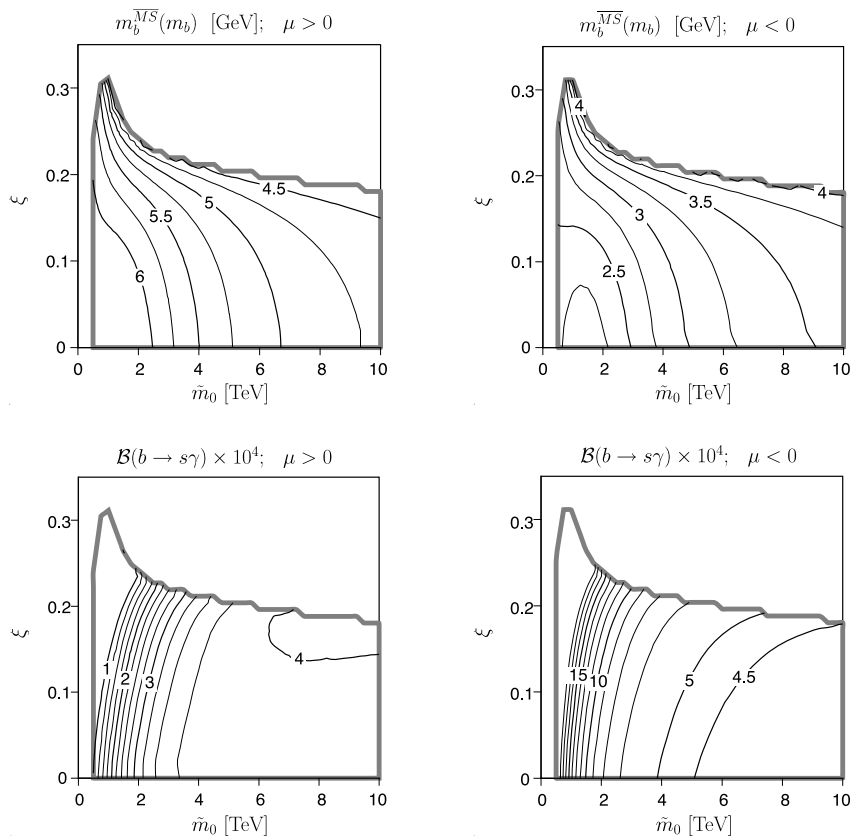
Figure 9: The same as figure 8 but for $\tilde{m}_0$–ξ parameter space. The SUSY-breaking parameters are set as $M_{1/2} = 300\,\text{GeV}$, $A_0 = 0$, and $N = 2.5$ at the GUT scale. In each figure, the narrow left region is excluded by the LSP scalar tau lepton and the upper side is ruled out the experimental mass bounds on CP-odd neutral Higgs boson, charginos, and neutralinos.

obtain an enough suppression of Δ_b, and therefore lighter chargino and neutralinos contain non-negligible higgsino components. For heavier scalars ($\tilde{m}_0 \gg$ a few TeV), the lighter chargino and neutralinos can be either gaugino-like or higgsino-like. The prediction of CP-odd neutral Higgs mass is correlated with the initial scalar mass $\tilde{m}_0$ as $M_A \sim 0.1\tilde{m}_0$. If one considers the case that $\tilde{m}_0$ is less than a few TeV, the charged Higgs loop A_{H^+} gives a sizable contribution to the total $b \rightarrow s\gamma$ decay width. Also the chargino loop $A_{\tilde{\chi}^+}$ gives a non-negligible contribution. We find that, in the parameter region consistent with the experimental bounds, A_{H^+} and $A_{\tilde{\chi}^+}$ are of the same order and have opposite signs. For $\tilde{m}_0 \gtrsim 2\,\text{TeV}$, the cancellation of partial amplitudes is enough to satisfy the experimental constraint from $b \rightarrow s\gamma$ decay.

As for the negative μ case, the above cancellation of diagrams cannot be obtained, though PQ and R symmetric spectrum is still available and the $b \rightarrow s\gamma$ branching ratio is suppressed by assuming a relatively large scalar mass $\tilde{m}_0$. In the region that the bottom quark mass prediction is within the experimental range, the PQ and R symmetries imply light charginos which lead to an enhanced contribution to the $b \rightarrow s\gamma$

branching ratio. For example, for $m_t^{\text{pole}} = 178\,\text{GeV}$ as in figure 9, the threshold correction must be more suppressed than the $\mu > 0$ case, which requires a tiny value of μ parameter. That makes it difficult to predict the observed value of bottom quark mass without introducing very large scalar masses. When the top quark mass is taken to be a smaller value, the bottom quark mass is increased and then the required value of $|\mu|$ becomes larger. Furthermore a smaller value of the top quark mass increases scalar quark masses for a fixed value of $\tilde{m}_0$ and hence the $b \to s\gamma$ constraint is relaxed. We find, for example, that $m_t^{\text{pole}} = 172.7\,\text{GeV}$ [28] is consistent with $N = 3$ and $\tilde{m}_0 \gtrsim 6\,\text{TeV}$.

Finally we comment on the lepton flavor violating decay of charged leptons. It is known in the Yukawa unification scenario [45] that a large value of $\tan\beta$ enhances the decay amplitudes of charged leptons such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$. However in the present analysis we do not specify small elements of lepton Yukawa couplings which control the generation mixing of charged leptons. Furthermore the generation mixing from Yukawa couplings is affected by the structure of right-handed neutrino Majorana mass matrix, the detail of which is also irrelevant to the present analysis of EWSB. For these reasons the prediction of flavor-violating rare decay of charged leptons is not under control and could easily be consistent with the current experimental bounds.

To summarize, we have found that the right-handed neutrino couplings induce new types of radiative EWSB scenarios. The low-energy superparticle mass spectrum is significantly modified by sizable contributions of neutrino couplings in the RG evolution down to the decoupling scale of right-handed neutrinos. In particular, the PQ and R symmetric spectrum is available to achieve the observed values of bottom quark mass and $b \to s\gamma$ branching ratio with heavy scalars of a few TeV. The μ parameter can also take a small value as $|\mu| \lesssim 0.15\tilde{m}_0$ and lighter chargino and neutralinos contain a sizable amount of higgsino component, which may be cosmologically favorable in that the LSP provides dark matter component of the present universe [46]. If the top quark mass is taken to be smaller, the phenomenological requirements are more easily satisfied.

5. SO(10) unification with large lepton mixing

The recent experimental results of solar and atmospheric neutrinos have revealed that there exist large flavor mixings in the lepton sector, while the corresponding mixing angles in the quark sector are observed to be small. In typical GUT scenarios, quarks and leptons are unified into a large multiplet and consequently their Yukawa couplings satisfy some simplifying relations. Thus the observed difference between the flavor structures of quarks and leptons is confusing but exciting issue in particle physics. For example, in the minimal SU(5) GUT scenario where down-type quarks and lepton doublets belong to the same multiplets 5^*, their Yukawa couplings are related as $Y_d = Y_e^T$ at the GUT-breaking scale. If Y_e has large off-diagonal elements which are suitable for suggested large generation mixing, it is naturally expected that the quark mixing matrix also contains large angles which are not compatible with the observation.

One of the attractive approaches to this problem is to consider the following generation asymmetric form of Yukawa couplings:

$$Y_d \simeq Y_e^{\mathrm{T}} \propto \begin{pmatrix} & & \\ & & \\ & a' & a \end{pmatrix}, \qquad (5.1)$$

where a, a' are of a similar order and the other blank entries are small compared to a and a'. The similarity between Y_d and Y_e^{T} is a consequence of GUT gauge symmetry such as SU(5) or larger unified group. This asymmetric form of Yukawa couplings is referred to as the lopsided form in the literature [15 – 20].[1] A key ingredient of lopsided mass matrices is that the observed large leptonic 2-3 generation mixing is explained by dominant two elements in the charged-lepton Yukawa matrix with $a \simeq a'$, while preserving small quark generation mixing because only right-handed down-type quarks are largely mixed and it does not contribute to the physical quark mixing. Various types of GUT scenarios with lopsided mass matrices have been studied. In dynamical models based on the SO(10) group, some non-minimal field contents are involved to realize the lopsided form of Yukawa couplings. For example, the MSSM matter and Higgs fields do not have ordinary high-energy origins such as 16_i and 10_H adopted in the previous sections. This possibility has also been used to construct realistic models with larger unified symmetry than SO(10).

In this section, we study phenomenological issues such as the radiative EWSB in the SO(10) unification which accommodates lopsided mass matrices for neutrino physics. We typically consider the following form of Yukawa couplings at the GUT scale:

$$Y_u = Y_\nu = \begin{pmatrix} & & \\ & & \\ & & y_G \end{pmatrix}, \qquad Y_d = Y_e^{\mathrm{T}} = \begin{pmatrix} & & \\ & & \\ & y' & y_G \cos\theta \end{pmatrix}, \qquad (5.2)$$

where blank entries in each matrix are small compared to the filled entries. The near-maximal atmospheric neutrino mixing is explained by assuming that y' is of similar order to $y_G \cos\theta$. In the following analysis, we simply take $y' = y_G \cos\theta$, leading to the maximal 2-3 mixing angle from the charged-lepton sector that is the central value of the current experimental data [14].

It may be instructive here to illustrate a dynamical explanation of the existence of angle θ in the Yukawa matrix (5.2). A crucial observation for lopsided matrix form is the multiplicity of 5^* components in the theory. That is, while right-handed down quarks and lepton doublets are combined into 5^* representations of SU(5), there is no way to identify to which multiplet this 5^* should be embedded in larger symmetry than SU(5). In fact there are several sources of 5^* in SO(10) theory; 10, 16, 120 representations, etc. The simplest case for realizing lopsided generations is to suppose the second and third generation 5^*'s have different origins in more fundamental theory like SO(10). For example, in an explicit SO(10) model [18], the second generation fields in 5^* come unusually from

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[1]A systematic analysis has recently been performed in [47] for asymmetric forms of quark and lepton mass matrices taking account of generation mixing and neutrino physics.

a decuplet 10 of SO(10). In this case, low-energy down-type Higgs field should be a mixed state of 10 and 16 Higgs multiplets, otherwise several fermions become massless. The angle θ parametrizes the degree of such Higgs mixing; the down-type Higgs $5^*_{H_d}$ is composed as $5^*_{H_d} = 5^*(10_H)\cos\theta + 5^*(16_H)\sin\theta$ where $5^*(10_H)$ and $5^*(16_H)$ are the anti quintuplets contained in 10_H and 16_H Higgs fields, respectively. The angle θ is dynamically controlled in terms of mass parameters in GUT Higgs potential. An SO(10) invariant superpotential term $f16_3 16_H 10$ gives a lopsided matrix with $y' = f\sin\theta$. It is interesting to notice that such 5^* flipping just corresponds to the $SU(2)_R$ rotation in SO(10) or higher theory. In the minimal SO(10)-type unification, the $SU(2)_R$ invariance in the Higgs sector is only weakly violated by small couplings, which makes the radiative EWSB difficult to be achieved. However in the present case, $SU(2)_R$ is strongly broken by the mixing of two types of 5^*'s, and the radiative EWSB is expected to be easily made successful. A similar analysis may also be performed for GUT models based on higher gauge groups than SO(10), e.g. the E_6 unification. An important difference appears in that, in the E_6 unification, the third-generation fields in 5^* have high-energy embedding into a 10-plet of SO(10) group. That implies (i) the matching conditions of the second and third-generation anti-quintuplets to low-energy multiplets are altered (i.e. exchanged from those in the SO(10) case), (ii) $\tan\beta \sim \frac{m_t}{m_b}\sin\theta \sim \mathcal{O}(1)$, and (iii) an additional D-term contribution arises from E_6 breaking down to $SU(5) \times U(1) \times U(1)'$. Therefore the induced low-energy phenomenology might be rather different. In particular, the above property (ii) could make the radiative EWSB much easier to occur.

5.1 SO(10) unification with asymmetrical Yukawa matrices

5.1.1 Bottom quark mass

Let us first examine the prediction of bottom quark mass in the SO(10) unification with the lopsided Yukawa matrices (5.2), in which we take for simplicity $y' = y_G\cos\theta$. The lopsided form of Yukawa matrices generally leads to the prediction of third-generation fermion masses quite different from that of the minimal SO(10)-type unification. As we have seen in the previous sections, the low-energy threshold correction Δ_b at SUSY-breaking scale plays a central role for predicting the bottom quark mass in Yukawa unified theory. In the present lopsided case, there is an additional important factor θ, which determines the mixture of high-energy Higgs fields and induces neutrino large generation mixing. That is, since the low-energy down-type Higgs in 5^* representation is now controlled by the angle θ, the predicted value of $m_b^{\overline{\rm MS}}(m_b)$ also has possible large dependence on θ. We show in figure 10 the prediction of bottom quark mass as the function of these two important factors Δ_b and θ. In the figures, we have $m_t^{\rm pole} = 178\,{\rm GeV}$, $\Delta_t = 0.03$, $\Delta_\tau = -0.02$, and $M_\nu = 10^{14}\,{\rm GeV}$. It is also found that $\tan\beta$ is another important quantity affected by the mixing parameter θ, since $\tan\beta$ is fixed by the prediction of tau lepton mass and the RG solution of tau Yukawa coupling is sensitive to θ.

It is seen from the figure that $m_b^{\overline{\rm MS}}(m_b)$ is increased for a larger value of θ. This is because the magnitude of bottom-quark and tau-lepton Yukawa couplings are suppressed by the factor $\cos\theta$. For a moderate value of $\tan\beta$, the bottom/tau mass ratio (without
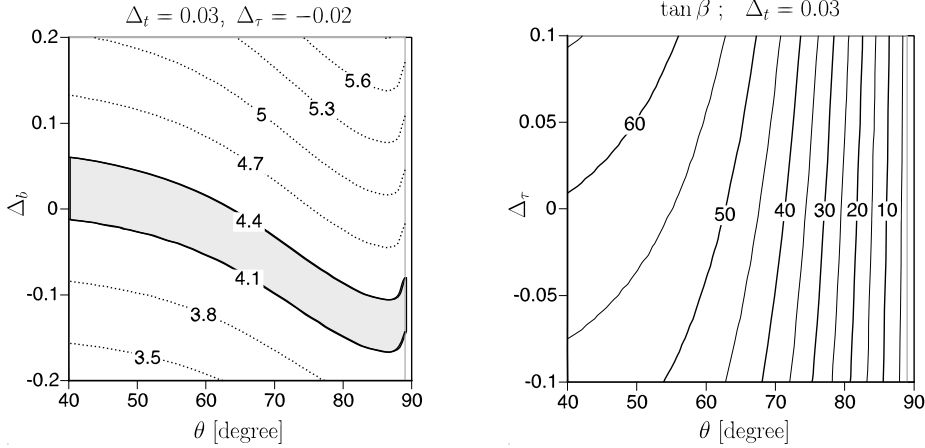
Figure 10: The predictions of bottom quark mass and $\tan\beta$ in SO(10) unification with lopsided mass matrices (5.2). The left figure shows $m_b^{\overline{\rm MS}}(m_b)$ as the function of mixing angle θ and SUSY threshold correction Δ_b. The right figure shows typical value of $\tan\beta$ in this model. Here we take input parameters the same as in the previous figure 1 as well as $m_t^{\rm pole} = 178\,{\rm GeV}$ and $M_\nu = 10^{14}\,{\rm GeV}$. The low-energy threshold corrections are set as $\Delta_t = 0.03$ and $\Delta_\tau = -0.02$ in the left figure and $\Delta_t = 0.03$ in the right one.

threshold corrections) is reached in the RG evolution down to low-energy regime to a larger value than that in large $\tan\beta$ case. This is because the Yukawa dependent contributions are reduced in the RG evolution of Yukawa couplings for a fixed value of $m_t^{\rm pole}$. With this behavior, a negative threshold correction Δ_b is required for $\theta \gtrsim 60°$ (see figure 10). Therefore a negative value of μ parameter is preferred in this model. We explicitly checked that this qualitative behavior is unchanged by varying input parameters in appropriate range. For $\tan\beta \lesssim 3$, the top quark mass $m_t^{\rm pole}$ is decreased as $\sin\beta$ and y_G is consequently increased. Thus the above discussion is not applied to the case of near-maximal value $\theta \simeq 90°$. In the following analysis we do not consider such a small value of $\tan\beta$.

Notice also from the figure that the absolute value of threshold correction to bottom quark mass must still be smaller than its naively expected size. Thus the argument of PQ and R symmetries may be helpful for suppressing Δ_b and attaining the experimentally-allowed value of bottom quark mass similarly to the minimal SO(10)-type unification. However there is an important difference between them; in the present model, the desired suppression factor of threshold correction Δ_b and also $\tan\beta$ have significant dependences on θ, which induces neutrino large mixing. In particular, even a bit large value of $|\Delta_b|$ is possible for small $\cos\theta$. Thus a relatively weaker PQ and/or R symmetric low-energy spectrum can be consistent with the experimental bound on bottom quark mass rather than the minimal SO(10)-type unification.

5.1.2 Radiative EWSB and PQ, R symmetric limits

To determine the complete structure of SUSY-breaking parameters at the GUT scale, we take the following simple assumptions: (i) the theory is just the MSSM with right-handed

neutrinos below the GUT-breaking scale. (ii) the MSSM matter fields, except for 5_2^*, come from 16_i ($i = 1, 2, 3$). (iii) the matter fields in 5_2^* originate from an additional matter 10. (iv) the up-type Higgs is included in 10_H, but the down-type one is a linear combination of $5^*(10_H)$ and $5^*(16_H)$ with a mixing angle θ. With this situation at hand, the matching conditions of MSSM SUSY-breaking parameters at the GUT scale are given by the following form:

$$m_{\tilde{Q}}^2(M_G)_{ij} = m_{\tilde{u}}^2(M_G)_{ij} = m_{\tilde{e}}^2(M_G)_{ij} = (m_0^2 + \Delta - D)\,\delta_{ij}, \tag{5.3}$$

$$m_{\tilde{d}}^2(M_G)_{11} = m_{\tilde{L}}^2(M_G)_{11} = m_0^2 + \Delta + 3D, \tag{5.4}$$

$$m_{\tilde{d}}^2(M_G)_{22} = m_{\tilde{L}}^2(M_G)_{22} = m_0^2 + \frac{4}{5}\Delta - 2D, \tag{5.5}$$

$$m_{\tilde{d}}^2(M_G)_{33} = m_{\tilde{L}}^2(M_G)_{33} = m_0^2 + \Delta + 3D, \tag{5.6}$$

$$m_{\tilde{\nu}}^2(M_G)_{ij} = (m_0^2 + \Delta - 5D)\,\delta_{ij}, \tag{5.7}$$

$$m_{H_u}^2(M_G) = m_0^2 + \frac{4}{5}\Delta + 2D, \tag{5.8}$$

$$m_{H_d}^2(M_G) = m_0^2 + \left(\frac{4}{5}\cos^2\theta + \sin^2\theta\right)\Delta + \left(-2\cos^2\theta + 3\sin^2\theta\right)D. \tag{5.9}$$

Here we have written down rather generic expressions for the boundary conditions, including (i) the usual (flavor-blind) universal scalar mass m_0^2 given at the gravitational scale M_P, (ii) the D-term contribution denoted by D, which potentially arises from the GUT symmetry breaking $SO(10) \to SU(5) \times U(1)$, and (iii) a radiative effect Δ generated via RG running from M_P down to M_G. If one neglects Yukawa-dependent contribution, Δ is given by

$$\Delta = \frac{45}{4b_{10}}\left[\left(1 - \frac{b_{10}g_G^2}{8\pi^2}\ln\frac{M_P}{M_G}\right)^{-2} - 1\right]M_{1/2}^2, \tag{5.10}$$

where b_{10} is the beta function for $SO(10)$ gauge coupling and $M_{1/2}$ is the $SO(10)$ gaugino mass parameter evaluated at the GUT-breaking scale. It is found from this expression that Δ always takes a positive value and independent of $M_{1/2}$ due to the unknown beta-function factor. For comparison to the analyses in the previous sections, we parametrize the scalar masses by $\tilde{m}_0$ and ξ defined as

$$\tilde{m}_0^2 \equiv \left.\frac{m_{10}^2 + m_{16}^2}{2}\right|_{D=0} = m_0^2 + \frac{9}{10}\Delta, \tag{5.11}$$

$$\xi \equiv \left.\frac{m_{10}^2 - m_{16}^2}{m_{10}^2 + m_{16}^2}\right|_{D=0} = \frac{-\Delta}{10m_0^2 + 9\Delta}. \tag{5.12}$$

In the following discussion, we use $\tilde{m}_0^2$ and ξ instead of the original parameters m_0^2 and Δ. It is noted that, contrary to the previous analyses, the parameter ξ is now limited in the range $-1/9 < \xi < 0$ because of the positiveness of m_0^2 and Δ. Such a bound is however not so strict but might be relaxed by including RG effects of GUT Yukawa couplings and/or by assuming negative m_0^2 [48]. We also assume for simplicity that the GUT-scale scalar

trilinear couplings are flavor universal, referred to as A_0. Thus the independent variables describing SUSY-breaking parameters at the GUT scale are

$$\tilde{m}_0, \ \xi, \ M_{1/2}, \ A_0, \ B_0, \ D. \tag{5.13}$$

In addition to these, the mixing parameter θ is an important factor for determining superparticle mass spectrum. As for the radiative EWSB, a smaller value of $\cos\theta$ suppresses the $Y_{d,\tau}$ effects in the RG evolution and makes $m_{H_u}^2$ lower than $m_{H_d}^2$ in the infrared. Consequently the experimental mass bound of CP-odd neutral Higgs boson is expected to be satisfied in a wider parameter region and the radiative EWSB is operative more easily than the ordinary Yukawa unification. That is contrasted with the previous result that, in the Yukawa unification, a difficulty in realizing the difference $m_{H_u}^2 < m_{H_d}^2$ excludes a large portion of parameter space.

To explicitly confirm such EWSB property and to examine the possibility of having PQ and R symmetries, we numerically solve the MSSM RG equations with right-handed neutrino couplings and evaluate the masses of physical particles in the electroweak symmetry broken vacuum;

$$M_A^2 \ = \ (g_{ms} + g_{md}\xi)\tilde{m}_0^2 + g_M M_{1/2}^2 + g_{\mathrm{AM}}A_0 M_{1/2} + g_A A_0^2 + g_D D - M_Z^2, \tag{5.14}$$

$$|\mu|^2 \ = \ (h_{ms} + h_{md}\xi)\tilde{m}_0^2 + h_M M_{1/2}^2 + h_{\mathrm{AM}}A_0 M_{1/2} + h_A A_0^2 + h_D D - \frac{M_Z^2}{2}. \tag{5.15}$$

Typical behaviors of the coefficients g's and h's are shown in figure 11 as the functions of mixing parameter θ. In the calculation, the input parameters are taken as the same as in the previous solutions (figure 7) for comparison and $m_t^{\mathrm{pole}} = 178\,\mathrm{GeV}$. The EWSB conditions are solved at the renormalization scale $Q = 1\,\mathrm{TeV}$. It is found from the figure that the coefficients g's in the RG solution (5.14) obviously depend on θ. In particular, g_{ms}, g_M, and g_D are rather sensitive and increase as θ. For $\theta \gtrsim 55°$, both the scalar (g_{ms}) and gaugino (g_M) effects become positive and therefore an R symmetric radiative EWSB is viable. This confirms the RG evolution behavior of Higgs mass parameters mentioned above. The CP-odd neutral Higgs mass squared is positive for a smaller value of $\cos\theta$. The D-term effect (g_D) is mainly controlled by the initial θ dependence of the D-term contribution to down-type Higgs mass parameter at the GUT scale.

In contrast to the uneven profile of g's, the coefficients h's in the RG solution (5.15) are rather insensitive to the mixing angle θ. This is understood from the EWSB conditions that $|\mu|$ is almost determined only by $m_{H_u}^2$ for a not-so-small value of $\tan\beta$; $|\mu|^2 \sim -m_{H_u}^2$, and also from the fact that θ controls only 5^*, i.e. the down-type Higgs mass $m_{H_d}^2$. As a result, in a wide range of θ, the RG solution $|\mu|^2$ is found to receive positive contributions from scalar (h_{ms}, h_{md}) and gaugino (h_M) terms (notice here that ξ only takes a negative value). The A_0-dependent contributions cannot be largely negative because $h_M, h_A > 0$. We thus find that only the D-term contribution is relevant for decreasing $|\mu|^2$ and realizing a PQ symmetric EWSB. The D-term effect (h_D) can be either positive or negative depending on the sign of D, but has little dependence on θ since the up-type Higgs mass is no connection with θ. To make $|\mu|$ small requires a positive value of D. However that makes the CP-odd
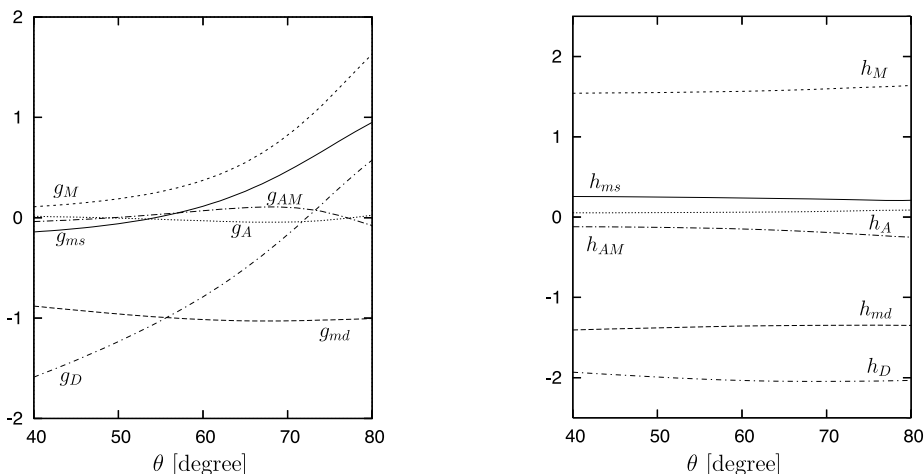
Figure 11: The θ dependences of the RG solutions (5.14) and (5.15) evaluated at the renormalization scale $Q = 1\,\text{TeV}$. Here we take input parameters as the same as in the previous figure 7 in addition to $m_t^{\text{pole}} = 178\,\text{GeV}$, $M_\nu = 10^{14}\,\text{GeV}$, and the SUSY threshold corrections $\Delta_t = 0.03$ and $\Delta_\tau = -0.02$.

neutral Higgs boson lighter if $\theta \lesssim 72°$. Thus it is not obvious whether approximate PQ and R symmetries are consistently realized in the superparticle spectrum in order for the third-generation fermion masses and the $b \to s\gamma$ constraint being acceptable.

For an illustrative purpose, let us focus on the exact limits of PQ and R symmetries. We have found in the above discussion that the PQ and R symmetries require a small $\cos\theta$ and a positive D term. To see the required values of D and θ, it is useful to examine the following functions:

$$\hat{D}_{\text{PQ}}(\theta) \;=\; \frac{-1}{h_D}(h_{ms} + h_{md}\xi), \tag{5.16}$$

$$\hat{M}_A^2(\theta) \;=\; g_{ms} - \frac{h_{ms}}{h_D}g_D + \Big(g_{md} - \frac{h_{md}}{h_D}g_D\Big)\xi, \tag{5.17}$$

where $\hat{D}_{\text{PQ}}$ satisfies the exact PQ symmetric equation $|\mu| = 0$ in the R symmetric limit and $\hat{M}_A^2$ denotes the CP-odd neutral Higgs mass (normalized by the scalar mass $\tilde{m}_0^2$) evaluated in these symmetric limits. Figure 12 shows the solutions $\hat{D}_{\text{PQ}}$ and $\hat{M}_A^2$ as the functions of θ and ξ. While $\hat{M}_A^2$ has large θ dependence, $\hat{D}_{\text{PQ}}$ is not so changed with θ. We find in the figure that the PQ and R symmetric EWSB with a positive mass squared of CP-odd neutral Higgs boson is achieved for $0 < D \lesssim 0.2\tilde{m}_0^2$ and $\theta \gtrsim 55°$.

To summarize the discussion about the radiative EWSB and the mixing angle dependence of bottom quark mass, there are two different options available; One is the case $\theta \gtrsim 60°$ with a negative μ and the other is $\theta \lesssim 60°$ with both signs of μ. The former is consistent with PQ and R symmetries and the suppression of large threshold correction Δ_b to bottom quark mass is obtained to have experimentally allowed bottom quark mass. In the latter option, only the R symmetric mass spectrum is possible but the suppression of
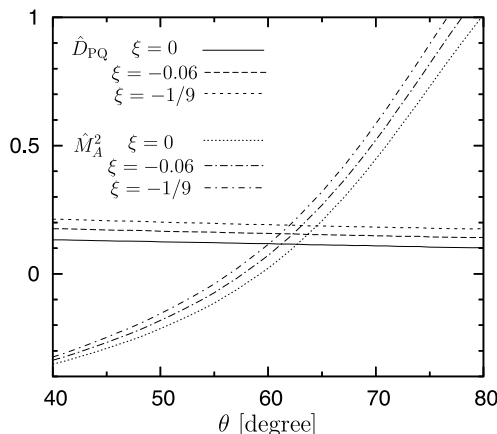
Figure 12: The solutions $\hat{D}_{\mathrm{PQ}}$ and $\hat{M}_A^2$ for the exact PQ and R symmetric limits (normalized by the scalar mass $\tilde{m}_0^2$). In this figure we take input parameters the same as in the previous figure 11.

the threshold correction would be still viable. To see these issues more explicitly, we will turn to the numerical parameter analysis in the next section.[2]

5.1.3 Parameter space analysis

As we have found in the previous subsection, there are two types of solutions for the radiative EWSB conditions and an enough suppression of excessive threshold correction to bottom quark mass. They are parametrized by the angle θ which induces neutrino large generation mixing. The solutions are divided into two regions, $\theta \gtrsim 60°$ and $\theta \lesssim 60°$, depending on whether PQ symmetric spectrum is viable or not.

Before proceeding to the numerical analysis, it is worth discussing the $b \to s\gamma$ process qualitatively. The $b \to s\gamma$ constraint is also affected by the angle θ through the initial values of couplings and RG evolution. As clarified in the analysis of Yukawa unification, for a negative μ, the $b \to s\gamma$ rare process generally gives stronger constraints on SUSY-breaking parameters than for a positive μ. Such qualitatively behavior of the branching ratio can also be applied to the present model as long as $\tan\beta$ is not so small. Thus the region $\theta \lesssim 60°$ with a positive μ is expected to easily avoid the $b \to s\gamma$ constraint than the other regions in which a negative μ is required to obtain the successful prediction of bottom quark mass. It is also noted that the gluino loop contribution to $b \to s\gamma$ decay is important [50]. This is due to the lopsided form of mass matrices and the non-universality

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[2]An important point in the analysis in section 5 is to consider generation asymmetric values of Yukawa couplings for charged leptons. On the other hand, boundary values of SUSY-breaking parameters depend on explicit dynamics above the GUT scale such as the Higgs content. For example, when 126-plet Higgs is adopted instead of 16 Higgs [49], $m_{H_d}^2$ is modified from eq. (5.9): $\sin^2\theta\Delta \to \frac{20}{9}\sin^2\theta\Delta$ and $3\sin^2\theta D \to -2\sin^2\theta D$ (θ now means the mixing of 10 and 126 Higgses). These modifications are not essential for the parameter space analysis and the qualitative behavior is not altered: The constraints from μ parameter are unchanged. In the PQ symmetric limit, a positive D is required which lowers M_A^2. That favors a region with larger gaugino mass effect, i.e. a larger value of mixing angle θ. For a smaller value of θ (with departure from the exact PQ symmetry), a larger negative value of g_D (the D-term coefficient in M_A^2) implies a negative D and relaxes the constraints from M_A^2 where smaller values of SUSY-breaking mass parameters are allowed.
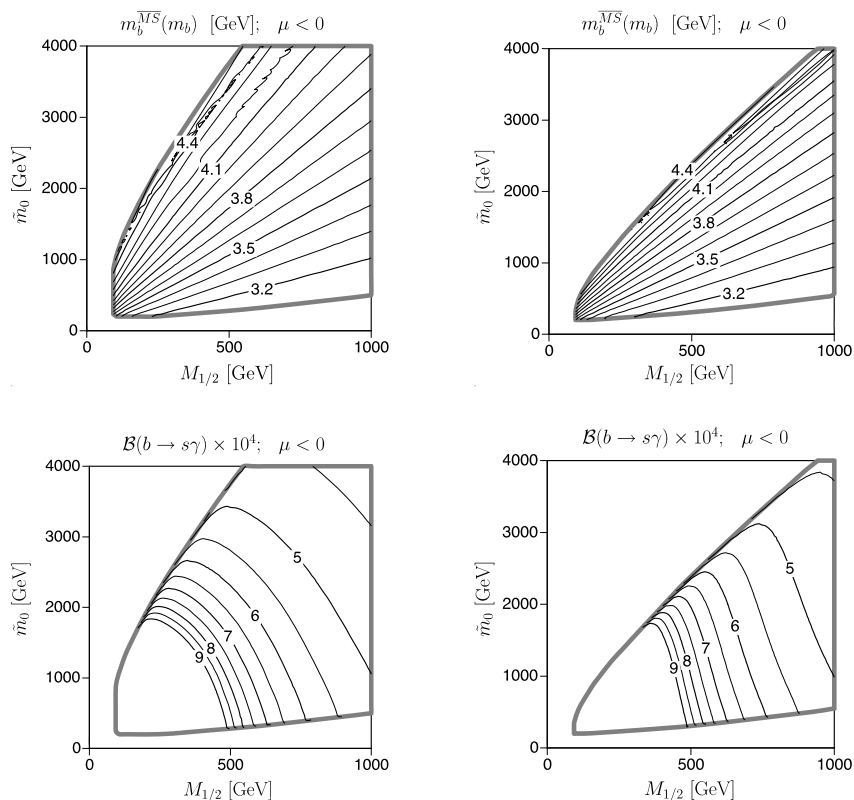
Figure 13: The $M_{1/2}$–$\tilde{m}_0$ parameter space consistent with the radiative EWSB conditions, the experimental mass bounds of superparticles, and the requirement of neutral LSP in SO(10) unification with the lopsided mass matrices. The bottom quark mass $m_b^{\overline{MS}}(m_b)$ and the $b \to s\gamma$ branching ratio are also shown in the figures. Here we take input parameters as the same as in the previous figure 4 in addition to $M_\nu = 10^{14}\,\text{GeV}$ and $\theta = 65°$. The scalar mass parameters are $\xi = 0$, $D = 0.1\tilde{m}_0^2$ in the left-sided figures and $\xi = -1/9$, $D = 0.2\tilde{m}_0^2$ in the right-sided ones. In each figure, the narrow right-bottom region is excluded by the scalar tau LSP and the left-upper side is ruled out by the current mass bounds of charginos and neutralinos.

of SUSY-breaking scalar masses. We now have a large 2-3 mixing of right-handed down quarks and flavor-dependent mass parameters of right-handed scalar down quarks. These flavor dependences cannot be rotated away by field redefinition and some imprint may appear, for example, as a large generation mixing of scalar down quarks. Furthermore the mass difference of up and down-type scalar quarks is induced by a negative value of D which mass difference is known to enhance the $b \to s\gamma$ amplitude via the gluino diagram $A_{\tilde{g}}$.

First we analyze the region of a small value of $\cos\theta$ $(\theta \gtrsim 60°)$. Such a large θ raises the tree-level bottom quark mass as seen in figure 10 and accordingly the negative sign of μ parameter is required. The threshold correction must be roughly $|\Delta_b| \sim \mathcal{O}(0.1)$ which is smaller than its naively expected size. Thus the approximate PQ and R symmetries are useful to obtain the experimentally allowed bottom quark mass. In figure 13, we show the $M_{1/2}$–$\tilde{m}_0$ parameter space consistent with the EWSB conditions, the experimental mass

bounds on superparticles, and the requirement that the LSP is charge neutral. In the figures, the predictions of bottom quark mass $m_b^{\overline{\mathrm{MS}}}(m_b)$ and the $b \to s\gamma$ branching ratio are shown in the allowed parameter regions. As an example, we set $\theta = 65°$ and ξ has the maximal value ($\xi = 0$) in the left-sided figures and the minimum value ($\xi = -1/9$) in the right-sided ones. In both cases, we obtain $\tan\beta \simeq 45$. In each figure, the narrow right-bottom region is excluded by scalar tau lepton being the LSP and the left-upper side is ruled out by the current mass bounds of charginos and neutralinos. In both extreme cases $\xi = 0$ and $-1/9$, we find that there exist the parameter spaces which reproduce the observed bottom quark mass, since a positive D term suppresses $|\mu|$. Also the $b \to s\gamma$ branching ratio is found to be within the experimentally observed range. This is achieved with a few hundred GeV gaugino masses and a few TeV scalar masses in both parameter spaces. Such a heavy scalar spectrum is due to the negative sign of μ. In the parameter region in figure 13, a positive D increases the bottom scalar quark mass compared to those of up-type ones. Thus the gluino-loop contribution to $b \to s\gamma$ process becomes smaller than the chargino-loop contribution.

We have found in this first case that, in the parameter space where the bottom quark mass and the $b \to s\gamma$ branching ratio are in agreement with the experimental range, superparticle mass spectrum exhibits approximate PQ and R symmetries. In particular, lighter chargino and neutralinos contain significant components of higgsinos. Moreover the mass bound of CP-odd neutral Higgs boson does not lead to strong constraints on the GUT-scale mass parameters. In fact, the CP-odd neutral Higgs mass takes as large as $2\,\mathrm{TeV}$ in the allowed parameter regions. These features are quite different from those obtained in the minimal SO(10)-type unification.

Next let us turn to studying another case with $\theta \lesssim 60°$. Both signs of μ parameter are allowed in this case. In the following, we take a positive μ which is advantageous to avoid the $b \to s\gamma$ constraint. The threshold correction to the bottom quark mass must be again small. Since the PQ symmetry is largely violated in this region, to reduce the size of $|\Delta_b|$ is obtained, for example, by large scalar masses $\tilde{m}_0 \gg M_{1/2}$. We show in figure 14 the bottom quark mass $m_b^{\overline{\mathrm{MS}}}(m_b)$ and $\mathcal{B}(b \to s\gamma)$ as the functions of scalar mass parameters $\tilde{m}_0$ and D. In each figure, the left and bottom regions are excluded by the charged LSP and the right one is excluded by the mass bound of CP-odd neutral Higgs boson. From figure 14, one can see that the prediction of bottom quark mass is decreased as $\tilde{m}_0$ and also has a large D dependence. This is because the RG evolution and EWSB conditions leads to a suppressed $|\mu|$ with raising D, and then the threshold correction $|\Delta_b|$ tends to be small. Therefore a large size of negative D-term contribution is disfavored. For example, the mass eigenvalue of the lighter scalar bottom quark has a significant dependence on D and is minimized around $D \simeq -0.25\tilde{m}_0^2$. From such property, the bottom quark mass and the $b \to s\gamma$ branching ratio are highly enhanced through the gluino-scalar bottom diagrams (the peaks in figure 14).

We have found in this second case that large scalar masses and small D-term contribution are suitable for low-energy phenomenology. The $b \to s\gamma$ constraint is easily evaded with enough heavy scalar quarks to suppress the superparticle contributions. While the
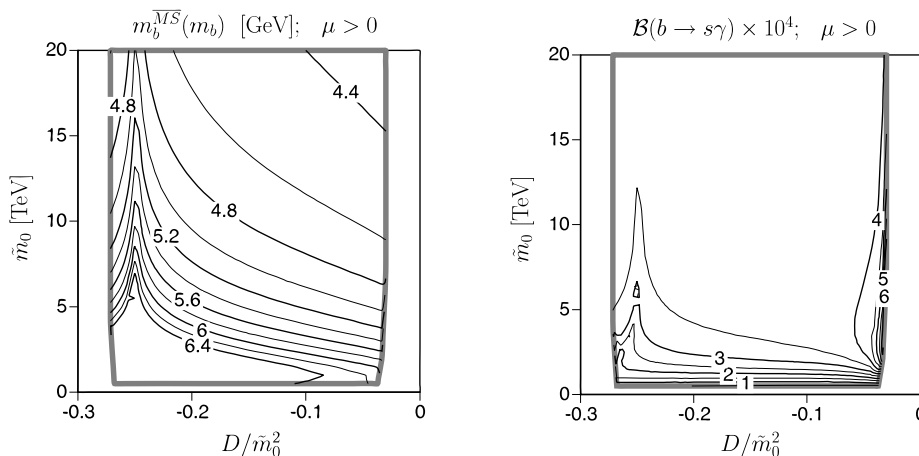
Figure 14: The same as figure 13 but for the parameter space of D-term contribution and scalar masses. Here we set $\theta = 50°$, $\xi = 0$, $M_{1/2} = 250\,\text{GeV}$, and $A_0 = 0$. In each figure, the left and bottom regions are excluded by the LSP being charged and the right one is excluded by the mass bound of CP-odd neutral Higgs boson.

scalars become heavy, the gauginos are relatively light and R symmetric spectrum is obtained with which the threshold correction to bottom quark mass is suppressed. Due to a large value of $|\mu|$, the lightest chargino and neutralino are gaugino-like. The CP-odd neutral Higgs is much lighter than the scalar quarks due to a small size of D. The charged Higgs boson tends to be lighter but can easily be made up to a few TeV and the charged Higgs contribution A_{H^+} is suppressed.

Finally we comment on the lepton flavor violating decay of charged leptons. As in the SO(10) unification scenario in section 4, the $\mu \to e\gamma$ decay rate is not under control as long as Yukawa couplings for the first and second generations and right-handed neutrino Majorana masses are unspecified. On the other hand, the $\tau \to \mu\gamma$ amplitude is calculable and expected to be large due to the Yukawa-induced large mixing for explaining the atmospheric neutrino anomaly. In the minimal supergravity boundary conditions, the $\tau \to \mu\gamma$ decay rate is sometimes marginal to the current experimental upper bound [51]. We have however found in the present scenario that $\tan\beta$ can be lowered and scalar leptons are relatively heavy. That makes the constraints from lepton flavor violation rather weakened.

To summarize the results of SO(10) unification with the lopsided mass matrices (5.2), there are two types of allowed parameter regions classified by θ which is the mixing parameter of Higgs fields and control neutrino large generation mixing. The first region is defined by $\theta \gtrsim 60°$ which leads to the PQ and R symmetric radiative EWSB and the prediction of bottom quark mass is well within the experimental range. Such a large θ requires a negative value of μ. The $b \to s\gamma$ constraint is also avoided if scalar masses are a few TeV. The approximate PQ and R symmetries lead to the lightest chargino and neutralino being higgsino-like (possibly the LSP dark matter), that does not appear in the minimal SO(10)-type unification. In the other region, $\theta \lesssim 60°$, the R symmetric mass spectrum allows the prediction of bottom quark mass well within the experimental range,

while PQ symmetry is not realized. The μ parameter can be either positive or negative. The $b \to s\gamma$ constraint is satisfied with heavy scalar quarks. In this mass spectrum, only the gauginos are expected to be relatively light. A crucial difference between the minimal SO(10)-type unification and the present model is the twisting of 5^* fields which affects Yukawa couplings and SUSY-breaking parameters at the GUT scale. That strongly violates the $SU(2)_R$ symmetry and thus the radiative EWSB is operative easier than the minimal SO(10)-type unification. The degree of 5^* mixing also affects the prediction of bottom quark mass which restricts the sign of μ parameter. In the present scenario, a moderate value of $\tan \beta$ implies a negative value of μ.

5.2 Asymmetrical Yukawa matrices modified

5.2.1 Bottom quark mass

The SU(5) gauge symmetry implies the down and charged-lepton Yukawa matrices are exactly same; $Y_d = Y_e^{\mathrm{T}}$. However to reproduce the observed mass pattern including the first and second generations requires some violation of SU(5) symmetry in the Yukawa sector. A well-known example of symmetry-violating sources is the group-theoretical factor arising from Higgs fields in higher-dimensional representations such as 45 of SU(5) and 126 of SO(10). That introduces a relative factor -3 between down-quark and charged-lepton Yukawa couplings [52] (the factor 3 means the number of colors and the negative sign comes from the traceless property of irreducible representations).

Keeping this issue in mind, here we consider an example of lopsided form of mass matrices which does not respect the SU(5) symmetry, that is, $Y_d \neq Y_e^{\mathrm{T}}$. In this section, the following simplified form is assumed for the down-quark and charged-lepton Yukawa couplings at the GUT scale:

$$Y_d \;=\; y_G \cos\theta \begin{pmatrix} & & \\ & & \\ & \frac{-1}{3} & 1 \end{pmatrix}, \qquad Y_e = y_G \cos\theta \begin{pmatrix} & & \\ & & 1 \\ & & 1 \end{pmatrix}, \qquad (5.18)$$

where the blank entries are negligibly small compared to the filled entries. The large two elements in the charged-lepton Yukawa matrix are responsible for the atmospheric neutrino mixing and they are now assumed to be equal, which leads to the maximal mixing angle from the charged-lepton sector that is the central value of the current experimental data. Compared with the SU(5) symmetric lopsided form (5.2), the 3-2 element in Y_d now involves a relative factor $-1/3$. This factor may originate from, for example, higher-dimensional representations of Higgs fields or higher-dimensional operators effectively inducing Yukawa terms.

An important effect of group-theoretical factor appears in the bottom/tau mass ratio. With the modified asymmetrical Yukawa matrices (5.18) at hand, the initial mass ratio at the GUT scale is estimated as

$$\frac{m_b(M_G)}{m_\tau(M_G)} \;=\; \frac{\sqrt{5}}{3} \;\simeq\; 0.75. \qquad (5.19)$$
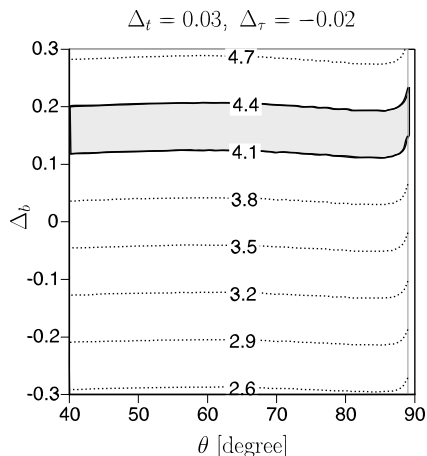
Figure 15: The prediction of bottom quark mass $m_b^{\overline{\text{MS}}}(m_b)$ in the SO(10) unification with the modified lopsided mass matrices (5.18). Here we take input parameters the same as in the SU(5) symmetric case (figure 10).

Therefore the RG evolution predicts a low-energy value of the bottom/tau mass ratio without including SUSY threshold corrections smaller than the previous SU(5) symmetric lopsided model. The modified bottom/tau mass ratio (5.19) indicates that the tree-level bottom quark mass at low energy is decreased from the case $Y_d = Y_e^{\text{T}}$ and now requires non-vanishing SUSY threshold correction Δ_b. In figure 15, we show the prediction of bottom quark mass as the function of Δ_b and θ, where the input parameters are taken as the same as in the previous figure 10. We find that the bottom quark mass is rather insensitive to θ and a sizable and positive Δ_b is needed to attain the observed bottom quark mass in a wide range of parameter space; $0.1 \lesssim \Delta_b \lesssim 0.2$. We numerically checked that this result is not changed qualitatively by varying other input parameters. The θ dependence of $\tan\beta$ is also examined and found to be almost the same as the previous SU(5) symmetric case (figure 10).

The above result, i.e. a positive Δ_b, implies that the μ parameter must be positive in the wide range of θ to reproduce the correct bottom quark mass. That is quite contrast to the SU(5) symmetric lopsided case $Y_d = Y_e^{\text{T}}$ with which a small $\cos\theta$ prefers a negative value of μ. An important point is that a positive μ parameter makes the cancellation possible among different $b \to s\gamma$ decay amplitudes via the charged Higgs boson and superparticles. The total branching ratio of the $b \to s\gamma$ rare process can be suppressed. The compatibility of a positive μ parameter and a relatively small value of $\cos\theta$ implies that, if one considers PQ and R symmetric mass spectrum, the $b \to s\gamma$ constraint is easily avoided than the SU(5) symmetric case. That is explicitly shown by a detailed analysis in the following subsection.

5.2.2 Radiative EWSB

As in the previous analyses, it is useful to solve the EWSB conditions about the CP-odd neutral Higgs mass M_A and the μ parameter. They are determined by the GUT-scale
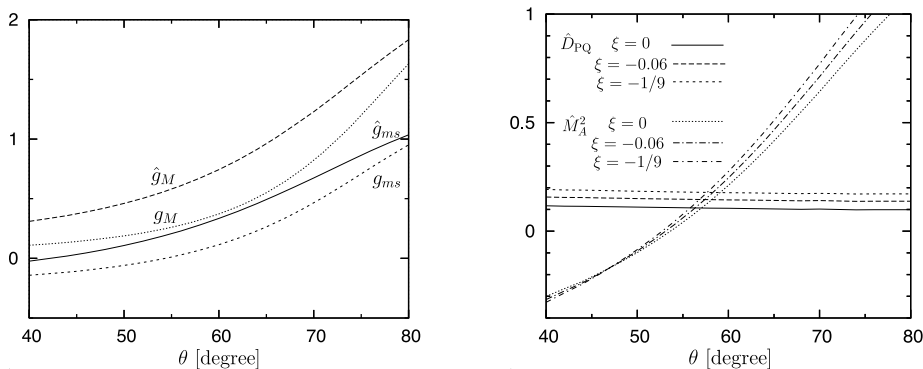
Figure 16: The left figure shows the RG solution for the CP-odd neutral Higgs mass, in particular, the scalar and gaugino pieces $\hat{g}_{ms}$ and $\hat{g}_M$ [for the parametrization, see eq. (5.14)]. For comparison, the corresponding coefficients g_{ms} and g_M are also presented. The right figure is the same as figure 12 but for the modified Yukawa matrices (5.18). In these figures, we take input parameters as the same as in figure 12.

SUSY-breaking parameters through the MSSM RG equations with right-handed neutrino couplings. The solutions therefore implicitly depend on the mixing angle θ which relates to neutrino large generation mixing. We here focus on the compatibility of successful radiative EWSB with approximate PQ and R symmetric mass spectrum. As has been discussed, these symmetries are favorable for obtaining acceptable bottom quark mass, $b \to s\gamma$ decay rate, and suitable amount of dark matter component of the universe.

The matching conditions for the MSSM SUSY-breaking parameters at the GUT-breaking scale are supposed to be the same as those in the previous SU(5) symmetric case.[3] The solutions to the RG equations and EWSB conditions are therefore almost similar to the previous ones (5.14) and (5.15). We here denote the coefficients in the present RG solutions as $\hat{g}$'s and $\hat{h}$'s corresponding to g's and h's in (5.14) and (5.15). We show in figure 16 the coefficients $\hat{g}_{ms}$ and $\hat{g}_M$ in the RG solutions for M_A^2. For comparison we also re-present the corresponding factors g_{ms} and g_M. The other $\hat{g}_x$'s and $\hat{h}_x$'s are found to have no sizable differences from corresponding quantities; $\hat{g}_x \simeq g_x$ and $\hat{h}_x \simeq h_x$ $(x \neq ms, M)$. As in the previous SU(5) symmetric case, $\hat{g}_{ms}$ and $\hat{g}_M$ have large θ dependence and increase as θ, since it directly controls the relative strength of Yukawa couplings. Thus a small value of $\cos\theta$ generally leads to a positive mass squared of CP-odd neutral Higgs boson. Moreover in the present model, the 3-2 element in Y_d is smaller than the SU(5) symmetric case (5.2). That makes $m_{H_d}^2$ larger in the infrared regime through smaller effects of Yukawa terms in the RG evolution of $m_{H_d}^2$. Consequently, the CP-odd neutral Higgs mass is easily

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[3]The existence of the factor $-1/3$ in the modified matrix Y_d requires some higher-representation fields or higher-dimensional operators in the Higgs sector. In the latter case, the MSSM matter and Higgs fields apparently have the same forms of couplings as in the SU(5) symmetric lopsided model, and all the matching conditions are unchanged. In the former case, however, higher-representation multiplets modify radiative effects. For example, if a 144-plet Higgs of SO(10) is adopted for the factor $-1/3$, only the matching condition for down-type Higgs mass is modified to $m_{H_d}^2(M_G) = m_0^2 + \left(\frac{4}{5}\cos^2\theta + \frac{17}{9}\sin^2\theta\right)\Delta + (-2\cos^2\theta + 3\sin^2\theta)D$. We found that the modification is so small that the results are unchanged almost quantitatively.

raised. This is numerically understood from the behavior of $\hat{g}_{ms}$ and $\hat{g}_M$ in figure 16; they are always larger than the corresponding g's for fixed values of θ and m_t^{pole}.

Unlike $\hat{g}$'s, the coefficients $\hat{h}$'s in the RG solution for $|\mu|^2$ are insensitive to θ and the results in the SU(5) symmetric case are equally applied to the present case. This implies that the PQ symmetric spectrum, that is, a small size of $|\mu|$, is achieved with a positive D term. The coexistence of approximate PQ symmetry and a well-lifted CP-odd neutral Higgs mass is allowed depending on the mixing angle θ. With the modified lopsided Yukawa couplings, we are able to have a smaller θ consistent with the experimental bound on M_A, as shown in figure 16 (the right figure). The figure shows that a weak bound $\theta \gtrsim 55°$ and a positive D-term contribution make PQ and R symmetric radiative EWSB available.

5.2.3 Parameter space analysis

It is now clear that a crucial discrepancy between the SU(5) symmetric and modified lopsided Yukawa matrices is the (signature of) threshold correction to bottom quark mass. This is a direct consequence of SU(5) breaking in the Yukawa sector, $Y_d \neq Y_e$. The modification of Yukawa couplings lowers the tree-level bottom quark mass and hence requires a positive μ, which in turn is consistent with PQ and R symmetric radiative EWSB. A more important implication of positive μ parameter is that an excessive $b \to s\gamma$ branching ratio can be reduced to be consistent to the observation with a cancellation among various partial amplitudes. In figure 17, we show the $M_{1/2}$–$\tilde{m}_0$ parameter space consistent with the EWSB conditions, the experimental mass bounds on superparticles, and the requirement that the LSP is charge neutral, i.e. the same as figure 13 but for different Yukawa forms and a positive μ parameter. In the figures, the predictions of bottom quark mass $m_b^{\overline{\text{MS}}}(m_b)$ and the $b \to s\gamma$ branching ratio are shown in the allowed parameter regions. As an example, we set $\theta = 65°$ and $\xi = 0$. The right-bottom region is excluded by the LSP scalar tau lepton and the left-upper side is ruled out by the current mass bounds of charginos and neutralinos.

From the left figure for the bottom quark mass prediction, we find that there exists a large parameter region to reproduce the observed bottom quark mass. Roughly speaking, the region around $\tilde{m}_0 \sim 2M_{1/2}$ is preferred and fat scalar particles are not needed. If one wanted to consider R symmetric spectrum $\tilde{m}_0 > 2M_{1/2}$, a larger D-term contribution should be included.

On the other hand, the right figure indicates that the constraint from $b \to s\gamma$ rare decay is much weakened and is readily made within the experimentally allowed range. We also find two separate regions consistent with the observation; $M_{1/2} \sim 200\,\text{GeV}$ and $M_{1/2} \gtrsim 400\,\text{GeV}$. In both cases, superparticles are relatively light, a few hundred GeV. This is a sharp contrast to the other scenarios discussed in this paper. Such a light spectrum is due to the positive sign of μ and the resultant cancellation of $b \to s\gamma$ decay amplitudes from the charged Higgs boson and superparticles. In the narrow parameter region between these two separate ones, the branching ratio tends to be too small since a relatively small gaugino mass enhances the chargino-loop contribution and the cancellation is too effective. We also note that, in figure 17 for $\mathcal{B}(b \to s\gamma)$, the left allowed region with a tiny gaugino
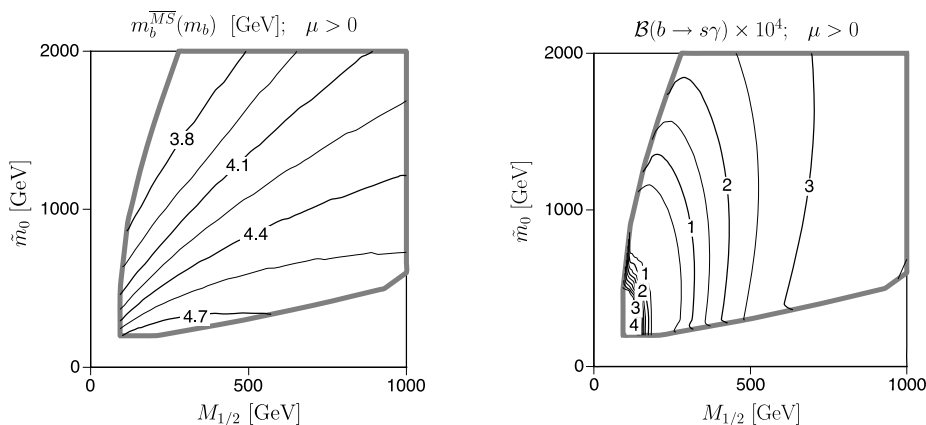
Figure 17: The $M_{1/2}$–$\tilde{m}_0$ parameter space consistent with the EWSB conditions, the experimental mass bounds on superparticles, and the requirement that the LSP is charge neutral, i.e. the same as figure 13 but for different Yukawa forms and a resultant positive μ parameter. The bottom quark mass $m_b^{\overline{MS}}(m_b)$ and the $b \to s\gamma$ branching ratio are also shown in the figures. Here we set $\theta = 65°$ and the GUT-scale SUSY-breaking parameters as $A_0 = 0$, $\xi = 0$, and $D = 0.1\tilde{m}_0^2$. In each figure, the right-bottom region is excluded by the scalar tau LSP and the left-upper side is ruled out by the experimental mass bounds of charginos and neutralinos. In this parameter space, we obtain $\tan\beta \simeq 45$.

mass is excluded by the experimental lower bound of the lightest Higgs boson mass, since the radiative corrections from the top sector [53] are not sufficient to meet the bound.

To summarize, in the SO(10) unification with modified lopsided form of Yukawa couplings, superparticles exhibit light and non-hierarchical mass spectrum to satisfy the observed values of bottom quark mass and $b \to s\gamma$ decay rate in the EWSB vacuum. This behavior is due to the fact that the threshold correction to bottom quark mass is needed to be positive and a bit large. That requires a positive μ parameter with which the $b \to s\gamma$ decay rate is made suppressed via diagram cancellations. The PQ and R symmetries are weakly attained in this model. The lightest neutralino and chargino consist of gaugino components, but possibly, a sizable amount of higgsino components is involved, which may be suitable for cosmological issues such as LSP dark matter. These features are quite different from the other scenarios in this paper and a detailed analysis is left to future work.

6. Summary

In this work we have investigated the low-energy phenomenology of supersymmetric SO(10) unification with neutrino effects suggested by its tiny mass scale and large generation mixing in the lepton sector. The analysis includes the radiative electroweak symmetry breaking, the third-generation fermion masses, and the flavor-changing rare processes.

In the general Yukawa unification with large $\tan\beta$, the observed fermion masses, especially the bottom quark mass, require suppressed threshold corrections at low-energy

decoupling scale of superparticles. The smallness of the corrections is ensured with super-particle mass spectrum which is approximately PQ and R symmetric, that is, the gaugino masses and supersymmetric Higgs mass parameter should be smaller than scalar masses. However in the minimal SO(10)-type unification without including neutrino couplings, the successful radiative EWSB leads to a large gaugino mass $M_{1/2} \gtrsim \tilde{m}_0$ to make the CP-odd neutral Higgs mass experimentally allowed. Consequently, low-energy SUSY-breaking parameters are strongly correlated to the gaugino masses, following which the threshold correction to bottom quark mass tends to be large and unacceptable.

Then we have included the effects of neutrino couplings in RG evolution down to the intermediate scale where right-handed neutrinos are decoupled. The newly introduced parameters are the neutrino Yukawa coupling of similar order of the other third-generation Yukawa couplings, the mass and trilinear couplings of right-handed scalar neutrinos. We have found that any of these three types of neutrino couplings is of great use for a successful EWSB. The parameter space $M_{1/2} \ll \tilde{m}_0$ gets allowed and the bottom mass threshold correction and the $b \to s\gamma$ decay rate are suppressed. The CP-odd neutral Higgs mass squared receives several positive contributions from the neutrino couplings in addition to the usual gaugino mass effect. Consequently the PQ and R symmetric superparticle spectrum can be consistent with the successful EWSB. For the positive μ case, excessive threshold corrections to bottom quark mass are suppressed for such type of superparticle spectrum. The $b \to s\gamma$ branching ratio is made within the experimental range, e.g. for $M_{1/2} = 300\,\text{GeV}$ and $\tilde{m}_0 \gtrsim 2.5\,\text{TeV}$. The μ parameter can also be made small and there appears the parameter region which accommodates the higgsino-like lightest neutralino. For the negative μ case, rather heavy scalars are inevitable because Δ_b should be highly suppressed. The constraint from $b \to s\gamma$ also requires heavy scalars $\tilde{m}_0 \gtrsim 10\,\text{TeV}$. If the top quark is taken to be lighter, the phenomenological constraints become satisfied by lighter scalars, $\tilde{m}_0 \gtrsim 6\,\text{TeV}$. In general, the low-energy superparticle spectrum is preferred to have hierarchical structure: light gauginos/higgsinos and heavy scalars are expected.

Finally we have taken into account the observed large generation mixing of neutrinos. In particular we have focused on the case that the maximal mixing between the second and third generations arises from the charged-lepton sector. An important factor is the parameter θ in high-energy Higgs sector which determines the neutrino large generation mixing. In the exact unification of down and charged-lepton Yukawa couplings, the tree-level bottom quark mass increases as θ, and a negative μ parameter is required. In addition, the CP-odd neutral Higgs mass is raised with a smaller value of $\cos\theta$. Thus the PQ and R symmetric radiative EWSB is possible with $\theta \gtrsim 60°$ and a positive D-term contribution. The observed bottom quark mass and the $b \to s\gamma$ constraint are easily satisfied with a few TeV scalar quarks. The lightest neutralino and chargino contain a sizable amount of higgsino components, which may be suitable for cosmological issues such as LSP dark matter. For $\theta \lesssim 60°$, while the PQ symmetric mass spectrum is not consistent with the positiveness of CP-odd neutral Higgs mass squared, the bottom quark mass is made within the experimental range only with help of R symmetry and relatively heavy scalars $\tilde{m}_0 \gtrsim 15\,\text{TeV}$. We have also examined the modification of SU(5) symmetric Yukawa couplings by introducing a group-theoretical factor for the masses of the second-generation fermions being prop-

erly reproduced. A crucial consequence of this modification is that the low-energy bottom quark mass without threshold correction is turned out to be reduced and a positive μ parameter is predicted. In this case, the $b \to s\gamma$ decay rate is made suppressed via diagram cancellations. Superparticles also exhibit light and non-hierarchical mass spectrum. These features are quite different from the other scenarios discussed in this paper.

In any case, our study has shown that the neutrino coupling effects induce new types of EWSB in SO(10) unification consistent with various experimental constraints. Physical implications of these scenarios such as predicted superparticle spectrum would be tested in the future experimental searches of supersymmetry.

Acknowledgments

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